COMPUTING CURRICULUM SUGGESTIONS FOR A WALDORF SCHOOL

Part 2 of 2
Lesson Plan Outlines and Content Overviews

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AWSNA

Table of Contents for Part 2

0. Focus of this Part 2.

When children relate what they learn to their own experience, they are interested and alive, and what they learn becomes their own.[1]

The lesson plan outlines and supporting content overviews below elaborate some parts of the curriculum in Part 1. The curriculum topics are assigned to 12th, 11th, 10th, and 9th as would be expected from Part 1. The manner in which the student's soul capacities are engaged can be broadly inferred from the thread of curriculum content; in addition, some detailed examples are given of how soul capacities are engaged in a balanced and dynamic way including the role of sleep.

The author's experience with this content is based on classes from the 2002-2003 school year at the Waldorf School of Saratoga Springs at which the computing course was confined to the 12th and 10th grades. The curriculum topics in Part 1 were configured as summarized in the table below – the numbers indicate the order of units in the two courses:

	Part 1 12th grade candidate topics	Part 1 11th grade candidate topics	Part 1 10th grade candidate topics	Part 1 9th grade candidate topics
Saratoga 12th grade course	3. Computing and human intelligence. 4. Good use of computers.	1. Algorithms. 2. Programming.	none	none
Saratoga 10th grade course	none	4. Morse code. 5. Logic gates.	1. Abacus. 2. Napier's rods. 3. Human computing team. 6. Electronic calculators.	none

Some anecdotal experience from teaching is reported below by past tense references to "Saratoga students". This was the author's first attempt to elaborate the curriculum in Part 1. Only some content has been elaborated to date. Of the three threads of content outlined in Part 1, the history thread is the least developed.

[1] Henry Barnes, *Waldorf Education … An Introduction*. Fair Oaks: Association of Waldorf Schools of North America.

1. 12^{th} grade: good uses of computers.

A "day" here is a 45 minute class – the 12^{th} grade Saratoga students had 4 such distinct classes per week.

Outline of lesson plans.

- Days One and Two
  - Synthesis of threads: For what tasks are computers a match - an approach to using computer products with judgment.
- Days Three and Four
  - Synthesis of threads: Moral action.
- Days Five and Six
  - Synthesis of threads: Guidelines concerning when to use a computer.

Overview of content for Days One and Two.

- For what tasks are computers a match? Good use of computers concerns the practice the student brings to their personal use of computers. The context for this work is a project that calls for use of commercial application software running on a PC and involves use of the Internet. This provides a "realistic" setting for establishing a practice of good use. Acting morally on computers requires an independent judgment as to whether the computer is a good match for the task in mind including whether the computer is being used for the good in relation to people. In relation to student soul capacity development, this question aims to help the student to school and develop and synthesize willing, feeling, and thinking required to form independent judgment:
  - Clarity of *thinking* informed by the student's understanding of what the computer can do based on work in grades 9, 10, 11, and 12.
  - The *feelings* that enter and inform the thinking. While the objective content of a judgment lies outside of feeling, for a student to be convinced of the correctness of a judgment then feeling must develop. Further, empathy for people enters into the judgment of whether the use is good in relation to people.
  - The *will* to act with commitment in relation to the outcome of the judgment.

  In relation to the outer world the student can develop experience and skills to judge whether and why a computer use is "good" for the task in mind, i.e., they can act morally upon computers. There is no intent to "moralize" here. By going through a process of making a considered judgment concerning the appropriateness of use for a computer the possible inclination to become will-bound is mitigated and the will can undergo a maturation process:

  > Willing is the same soul-activity as thinking, but willing is still a child. When it grows a little older, it becomes feeling, and when it is quite old it is thinking. The matter is made difficult by the fact that the different ages live together in our soul in these three activities.[2]

  This maturation can take place from one day to the next where sleep plays a critical role.[3] Periods of not doing are required to enable that which is fired up in the will to mature in the warmth of our heart as feeling and then further mature in our cool head as thoughts.
- The students proceed with an introduction to products such as Microsoft Explorer, Microsoft Outlook Express, and Microsoft Office applications. Skills of how to use and judgment concerning when to use are held together. The students work through diverse concrete tasks, e.g.,
  - Write a business letter combining text, a table, and graph of financial results. Has enough thinking been done before the letter is started? Should there be a handwritten draft first?
  - Obtain some credible facts for the letter from the Internet. Where is the Internet helpful and not helpful? How can sources be validated?
  - Perform an elaborate calculation for the business letter. Where is a spreadsheet useful, e.g., repetitive "what-if" computations and those dependent on parameters? Where can it be a distraction, e.g., when developing a personal feel for orders of magnitude?
  - Communicate the same information to many people. Where is email helpful, e.g., to overcome geographic or temporal separations?

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[2] Steiner, Rudolf. *The Balance in the World and Man, Lucifer and Ahriman*. North Vancouver: Steiner Book Centre, Inc., 1977. Page 33.

[3] This maturation can take also take place over long periods of time, such as the three initial seven year cycles of development which is reflected by the broad sweep of curriculum content progression.

- Identify events that compromise the integrity of the task, e.g., can the spellchecker replace proofreading, and is attention being drawn away from the task to how the products operate?

Overview of content for Days Three and Four.

- Where is the use moral, i.e., what is a computer "good" for? Are there certain tasks that a computer ought not to be made to do, independent of whether computers can be made to do them? Some focused questions that can be discussed open-endedly in a whole-class setting are as follows:
  - Unknown presence of a computer. A checkers playing program was tested[4] by playing against people at the Microsoft Network Gaming Zone Web site (www.zone.com). Based on the number of games played and the performance the machine was awarded a "Class A" rating – one level below the designation of "expert" following the United States Chess Federation rating system. Opponents were not told they were playing against a program, nor did any of them appear to guess that their rival was a program. Some people praised the ingenuity of their competitor. Should they have been told?
  - Unknown presence of a computer. Given that a company selling goods uses software to analyze incoming customer e-mail and compose response e-mails, should we be told that the response we receive is mechanized?
  - Replacing people. Given web-based software that can be used to emulate a psychotherapist to enable widely available and affordable "therapy" on the Internet, should this be made freely available?
  - Replacing people. Given a sufficiently sophisticated and affordable chess program that could beat most schoolchildren, should it be used to represent the school in chess matches rather than "less able" students?
  - Hacking. "White hat" hackers identify and publicly publish security flaws in products, such as Microsoft Windows. On the one hand this enables the manufacturers and the users to become aware of the problems and repair them; on the other hand, "black hat" hackers can use this information to attack vulnerable users before a repair has been put in place. Are white hat hackers doing the right thing?
  - Legal responsibility. Napster combined a centralized database service and distributed PC software to enable computers to exchange music; this was used by people to exchange music without paying copyright royalties to the music owners and makers. Does Napster have any responsibility for the copyright violation by its customers?
  - Human multitasking. Is it generally accepted that a person can walk and chew gum at the same time and do both "well". Mothers are known to be able to rock a baby and sew a torn vest at the same time and do both "well". However, is it possible to drive a car, hold a conversation on a mobile telephone, and check incoming e-mail at the dashboard and do all three "well"? What limits on multitasking should we consider when we work with computing machines? What capacities does a computer have that enable it to multitask, or appear to multitask, "well"? Do people have these capacities?
  - Unintended consequences. Computers can cause organizational forms to ossify. Weizenbaum[5] illustrates: "The belief in the indispensability of the computer is not entirely mistaken. The computer has become an indispensable component of any structure once it is so thoroughly integrated with the structure, so enmeshed with its vital substructures, that it can no longer be factored out without fatally impairing the whole structure. That is virtually a tautology. …. The computer was not a prerequisite to the survival of modern society in the post war period and beyond; its enthusiastic, uncritical embrace by the most "progressive" elements of American government, business, and industry quickly made it a resource essential to society's survival *in the form* that the computer itself had been instrumental in shaping." Discuss how to mitigate such a problem.
  - People's special needs. Consider that a brain-computer interface can be used to help people with severe paralysis to make physical actions.[6] The person intentionally modulates certain components of the electrical signals emitted by their brain, which is then recorded by an electrode-covered cap. A computerized signal-processing algorithm converts the signal into a command for a prosthesis. An example prosthesis consists of electrodes implanted under the skin in such a way as to choreograph movement in the muscles of the arm and hand so as to grasp a pen. Is this is a "good" use for computers?
  - Whither Artificial Intelligence. Consider Ray Kurzweil's view[7] that in the future: "We will also be able to scan a particular person -- let's say myself -- and record the exact state and position of every

[4] Bawa, Joanna. *Computers and Your Health*. Berkeley: Celestial Arts, 1996.
[5] Weizenbaum, Joseph. *Computer Power and Human Reason*. Penguin Books, 1984.
[6] Chase, Victor D. "Mind Over Muscles." *MIT Technology Review.* March/April 2000.
[7] Kurzweil, Ray. *The Age of Spiritual Machines.* Penguin Books, 1999.

neurotransmitter, synapse, neural connection, and other relevant details, and then reinstantiate that information into a neural computer of sufficient capacity. The person that then emerges in the machine will think that he is (and had been) me. He will say "I was born in Queens, New York, went to college at MIT, stayed in Boston, walked into a scanner there, and woke up in the machine here. Hey, this technology really works." Consider this in relation to limitations of computing such as algorithms and abstraction.

- Information overload. How should that be managed? Considerations include:
  - Distinction between "data", "information" as theoretical knowledge, and "knowledge" as practical personal knowledge.
  - More thoughtful sending to others.
  - Designing systems with the other person in mind.
  - The phenomenon of "peek-a-boo" information[8] that is hot today and gone tomorrow.
- Physical injury. A repetitive stress syndrome called carpel tunnel syndrome is increasingly common in corporations. It arises from typing and using a mouse at personal computers. How should this be mitigated?
- Limitations of information on the Internet. The web is a repository of finished thoughts of others. These valuable crystals correspond to the end product of thinking. Not all finished thoughts are available on the Web. Care is needed to evaluate the significance of information, e.g., credibility of authorship. What is the role of the Internet in supplying us with information?
- Chat rooms. Unlike face to face communication or telephone conversations you cannot be sure of the identity of who is on the other end of the line. What approach is appropriate?
- The rules of the day-to-day functioning of some commercial businesses are being given to computers. Enterprise Resource Planning software, originally designed for manufacturing plants, has been generalized to automate back office functions and increasingly front office and head office functions. The main leverage looked for by the company is reduced staffing costs. During remediation of Year 2000 computer issues[9], the risk management plans of some corporations revealed that the body of people who, by definition, are the corporation would not know how to run the business if the computers failed at the millennium. They had delegated away the knowledge of how to run the business. Is that OK?
- Computers sometimes mechanize missile launch decisions during war. What safeguards are necessary? The leverage looked for by military organizations is increased decision speed. In 1961 J.W. Forrester commented on developments on combat automation since his 1947 memorandum to the U.S. Navy "On the Use of Electronic Digital Computers as Automatic Combat Information Centers":

  > … one could probably not have found [in 1947] five military officers who would have acknowledged the possibility of a machine's being able to analyze the available information sources, the proper assignment of weapons, the generation of command instructions, and the coordination of adjacent areas of military operations …. During the following decade the speed of military operations increased until it became clear that, regardless of the assumed advantages of human judgment decisions, the internal communication speed of the human organization simply was not able to cope with the pace of modern air warfare. This inability to act provided the incentive.[10]

Overview of content for Days Four and Five.

- ♦ This curriculum content aims to help the student to school and develop their independent judgment in relation to using modern electronic computers by developing a positive practice for deciding whether computers are right for the task at hand. Accordingly, the guidelines are developed through brainstorming and reflection.
- ♦ Some candidate questions to evoke guidelines concerning when (as distinct from how) to use a computer are as follows:
  - Can the task be understood independently of the computer? If not, does the task needs penetrating more deeply or redefining? This step has two purposes:

[8] Postman, Neil. *Technopoly – The Surrender of Culture to Technology.* New York: Vintage Books, 1993.
[9] The Year 2000 (Y2K) issue is the result of systems with computers using two digits rather than four to represent the applicable year. As they make turn of the millennium based computations, they may interpret "00" as the year 1900 rather than the year 2000, or may not properly treat the year 2000 as a leap-year. Depending on how the date is made use of this could result in system failures or miscalculations.
[10] Forrester, J.W. in M. Greenberger, editor, *Managerial Decision Making in Management and the Computer of the Future.* MIT Press, 1962. Pages 52-53.

- First, it reinforces the intentionality of the operator around the task. This concerns the student keeping to the task and not being refocused on "getting the program to work" rather than completing the task.
- Second, the unavailability of the computer then becomes a more manageable problem and less a source of fear. The student should have a contingency plan as to how to finish the task if the computer breaks.

- Where would the computer be helpful with the thinking process? What care must be taken when delegating judgment?
  - Is delegation of a repetitive task OK? Perhaps to allow the operator to focus their capacities on more creative aspects of the task?
  - Is performing a computational task that would take "too long" for people to do OK? What is "too long"?
- Is there a plan to retain ownership of accuracy and meaning? For example, doing a "back of the envelope" calculation or performing the final and authoritative proofreading.
- Are there physical challenges to be met? A repetitive stress syndrome called carpel tunnel syndrome is increasingly common in corporations. It arises from typing and using a mouse at personal computers. What are technology based and other approaches to mitigating this problem?

2. 12th grade: computing and human intelligence.

A "day" here is a 45 minute class – the 12th grade Saratoga students had 4 such distinct classes per week.

Outline of lesson plans.

- Day One
  - History thread: examples of computing intelligence.
- Day Two
  - Algorithm thread: review of algorithms from 11th grade.
- Day Three
  - Algorithm thread: abstraction and its affect on meaning.
- Day Four
  - Algorithm thread: abstraction and its affect on meaning.
- Day Five
  - Algorithm and computing machine thread: review of electronic calculator limitations from 10th grade; accuracy and its relationship to meaning.
- Day Six
  - Algorithm thread: undecidability.
- Day Seven
  - Algorithm thread: undecidability.
- Day Eight
  - Algorithm thread: Turing test for "intelligence".
  - Algorithm thread: Eliza and successors.
- Day Nine
  - Synthesis of threads: Computing and human intelligence.

Overview of content for Day One.

- Nowadays, some people go as far as to regard powerful computers to have comparable, even superior, capacities to human thinking. The students bring some examples of the reasons why.
- Some leading edge examples are:
  - In May 1997 the Deep Blue machine became the first chess-playing computer ever to defeat a reigning world champion when it beat Garry Kasparov in a six-game match - two to Deep Blue, one to Kasparov, and three drawn. Kasparov had been unbeaten for fourteen years in match play. IBM's computer "Deep Blue" was a RISC System/6000 model weighing 1.4 tons and looks like a black refrigerator. In 1996, Kasparov had beaten Deep Blue in three out of six games with two draws. In 1989, Kasparov played Deep Blue's predecessor, Deep Thought, and won all the games. What had changed in the machine that lost in 1996: more powerful hardware, a more extensive chess database, and a program to change the parameters in between each game. Concerning power, new hardware had the ability to analyze 200 million moves a second, compared to Kasparov's estimated 3 moves a second. Thus in the three minutes usually allotted during tournaments for a chess player to make a move the machine could make 40 billion calculations of possible moves.
  - Experimental mathematics is using computer-aided proofs to augment the capabilities of human mathematicians. As examples:
    - Refutation of Euler's conjecture that at least N Nth powers would be required to sum another Nth power (three cubes, four fourth powers, etc.). Frye was able to find a counterexample by using a combination of conventional analysis and computer search[11] :

      (95,800)**4 + (217, 519)**4 + (414,560)**4 = (422,481)**4 =
      31,858,749,840,007,945,920,321

      Finding such a counterexample exceeds the capability of a human mathematician either doing mental arithmetic or using a paper and pencil.
    - The problem of whether four colors are sufficient to color any map. Haken and Appel attained the proof in 1976 using 1,200 hours of time on a then-powerful electronic computer[12]. The computer

[11] Stewart, Ian. *From Here to Infinity*. Oxford University Press, 1996. Page 36.
[12] Stewart, Ian. *From Here to Infinity*. Oxford University Press, 1996. Page 105.

searched through a large but finite number of possible maps that were specified in advance by Haken and Appel. Thereby the computer searched exhaustively for a counterexample. Human mathematicians have no way to check the computer's work since it would take many lifetimes.

Overview of content for Day Two.

- ♦ Algorithms from grade 11 are reviewed. See Section 4 below.

Overview of content for Day Three.

- ♦ While abstraction is used in other contexts, it is suggested the topic of abstraction be treated in relation to meaning. "What is the meaning of my computational results?" is a crucial question for a 12th grader to be working with in order to be able to self-direction to their life in a contemporary culture of pervasive computing.
- ♦ Partly due to its familiarity, and partly to warm the coldness of the topic of abstraction in the context of computing, the explicit process of abstraction is introduced through art. The students are asked to compare and contrast a series of four sculptures by Henri Matisse:

Back I, 1909, *Back II*, 1913, *Back III*, 1916-17, *Back IV*, 1930, are four bronze sculptures about 6 feet high and 4 feet wide by Henri Matisse. In the succession of works, the axis of the spine becomes the increasing focus of the composition. In 1908, Matisse wrote:

> In a picture, every part will be visible and will play the role conferred upon it, be it principal or secondary. All that is not useful in the picture is detrimental. A work of art must be harmonious in its entirety; for superfluous details would, in the mind of the beholder, encroach upon the essential details.[13]

The class arrives at a general view that an abstraction is a design technique that selects the essential aspects of a situation and conceals non-essentials. There is a conversation about the rise of abstraction in 20th century art and computing, both exploring forms that by their abstract nature carry some abstract intellectual content.

- ♦ The students are asked to design the essential items of data for a sales tracking system to determine salesperson wages based on commission. The system could be a paper one initially and then be mechanized as the business grows larger. The students are asked to make a list of attributes of a person and then a second list of which aspects are essential to be represented so their wages can be calculated monthly. The results are pooled and a common list starts to form:
  - ➢ Essentials for the abstraction "salesperson" are few, e.g.,
    - ❑ Name
    - ❑ Unique identifier, e.g., social security number, or unique company identifier
    - ❑ Dollar value of sales
    - ❑ Commission rate
  - ➢ Inessentials are many, e.g.,
    - ❑ Personal appearance
    - ❑ Medical history
    - ❑ Family configuration
    - ❑ Food preferences

  Then wages for (Name, Unique identifier) can be computed as Dollar value of sales x Commission rate. The unique identifier is required to disambiguate two identical names.

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[13] Barr, Alfred H. *Matisse: His Art and His Public.* Arno Pr., 1967.

- The students are asked to consider the design of a medical system to provide records for a person requesting medical treatment. The system could be a paper one initially and then be mechanized as the practice grows larger. The students are asked what aspects of a person are essential to be represented. The list starts to form:
  - Essentials for the abstraction "patient" are few, and different even for the same person from "salesperson":
    - Name
    - Unique identifier, e.g., social security number, or patient identifier
    - Medical history
    - Family medical history
  - Inessentials are many:
    - Dollar value of sales
    - Commission rate for sales …

  Then unique identifier can be used to access the record of data for Name.
- The intent for working with the soul capacities of a student on this day is as follows. A natural enthusiasm arises when writing a list of attributes of a person. These feeling*s* enter and permeate her will that is then raised to activity and engaged by the manual activity of writing down the attributes. Through the need to figure out the abstraction the student's will stirs her thinking. Thinking in turn enters her will to give intentionality to her nimble fingers, e.g., is the space of attributes to be drawn flat or patterned? Then an imaginative recollection of the facts stirs her feelings in relation to the hands-in work. These feelings are the fundament needed for the next step.

Overview of content for Day Four.

- The intent for this day is that, following sleep, pictures are present and available to the thinking. Reflections and discussions are possible that, through inspiration, bring to consciousness in the child the general rules that stand behind the observed examples from yesterday. This is the context within which the teacher presents the concepts of abstraction. Will, which can draw on the precedent of the manual work, enters her thinking to form, direct, and connect these thoughts. By deferring until this day the conjoining of concepts with the percepts from the previous day, a practice for "living thinking" is cultivated.
- The students reflect on whether the "salesperson" and "patient" abstractions have all the required essentials. The students identify desirable properties of an abstraction, e.g.,
  - Contains only what is essential.
  - Suggestively named.
- A conversation is held concerning the large number of abstractions in which one person can "participate". The students consider the importance of limiting the use of each abstraction to its original purpose. The diminished view of the person in the salesperson model should not be used for other purposes not intended by the model, e.g., to mechanize annual retention decisions based on value to the company – that could miss key contributions such as "mentorship to other employees" and outstanding long term financial contribution. Weizenbaum states the following:

  > … models embody only the *essential* features of whatever it is they intended to represent. If a model of an automobile is intended for wind tunnel tests, then the outside shape of the model car is important, but no seats nor any other interior furnishings of the real automobile need be present in the model. What aspects of reality are and are not embodied in a model is entirely a function of the model builder's purpose. But no matter what the purpose, a model, and here I am concerned especially with computer models of aspects of reality, must necessarily *leave out almost everything* that is actually present in the real thing. Hence what can be learned from manipulating the model is strictly limited *in every case*. Whoever knows and appreciates this fact, and keeps it in mind while teaching the use of computers, has a chance to immunize his or her students against believing or making excessive claims for much of their computer work. [14]
- Thus an abstraction can represent a problem by a model that can be manipulated algorithmically. Certain aspects of the problem complexity and detail are "abstracted away". The art of abstraction concerns the judgment, which improves with practice, of which details have little to no effect on the solution so we can have confidence that the model lets us deal with the essence of the problem. The actual problem is transformed into, in a sense "replaced" by, the model. Therefore the computing machine does not solve the actual problem – rather it solves a model of the problem. Then we must concern ourselves with how "good" is the model and how to interpret the "results" of the computation in relation to the original problem. Again this requires judgment. As we interpret the results do the inessentials still seem inessential? A picture that can be used to convey an often neglected step in the process of abstraction is shown below:

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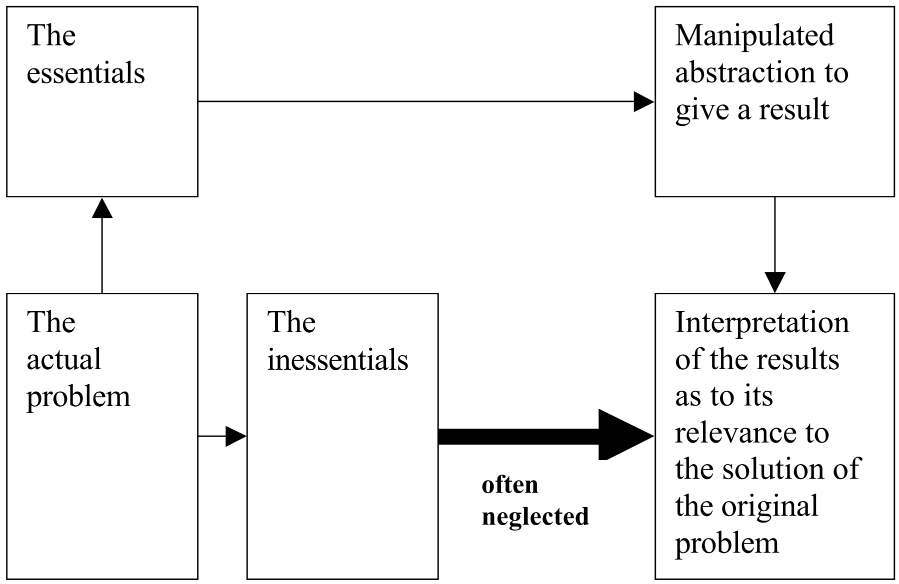
[14] Weizenbaum, Joseph. *Computer Power and Human Reason*. Penguin Books, 1984. Page xvii.



Overview of content for Day Five.

- Capacity and its limiting affects on precision and accuracy are reviewed by repeating the electronic calculator limitations exercise from 10th grade. See Section 5 below. A discussion is engendered concerning how accuracy relates to meaning, and how ownership over accuracy is retained by further scrutiny on our part, e.g., to determine if the result is reasonable and consistent with other observations. For example, our hypothetical power company from grade 10 work could have a quite wrong meaning associated with its precisely expressed but inaccurate numbers for power loss – they could spend millions of dollars trying to fix a problem that does not exist. Precise inaccuracy, with attendant loss of meaning, also appears in qualitative work, e.g., a software spell checker will approve the "correct" spelling of the wrong word such "on" instead of the intended "one". That example further illustrates how accuracy relates to meaning. As a quantitative example, if a certain experiment yielded the magnitude of the acceleration due to gravity as 980 meters per second per second (rather than the accurate 9.8), would it really feel right in relation to our everyday experience of falling objects? As a further exploration of the distinction between precision and accuracy, one can observe that in order to be more accurate in relation to a certain meaning, you might even have to be less precise, e.g., a researcher for Encyclopedia Britannica reviewed the question of whether the world's "largest" bear should be based on height or weight. "I leave it up to you," he wrote to the Britannica editors, "to decide whether we stay with the Kodiak … go with the polar bear, or fudge."[15] Thus we may be unable to decide unambiguously which is the world's "largest" bear, but that might be less important than having a full and rich appreciation of "large" bears.

Overview of content for Day Six.

- Undecidability, in the form of Emil Post's correspondence problem, is introduced as a "domino" play. The students work in pairs to create a warm social event and to help each other solve the problems. Each pair makes 8 domino-shaped slips of paper from one writing sheet. Domino types are made with strings of letters on their upper and lower halves. The task is to place dominos from the collection, repetitions allowed, such that the upper string and lower string created by concatenating the strings on the selected dominos are identical.
- A worked example teaches the approach: given {A/AA, ABA/B}, the students can use two instances of the first domino and one of the second domino to find the answer (A/AA, ABA/B, A/AA). Saratoga students asked for many clarifications of the rules, e.g., must each domino type be used (yes), can the dominos have their upper string and lower

[15] McCarthy, Michael J. "Britannica Plies the Info Highway." *Wall Street Journal.* April 22, 1999.

string swapped (no), can the dominos be used with just their upper string or just their lower string (no)? The Saratoga students explored the rules looking for flexibility and found little.

- Then other problems are given to solve, e.g.,
  1) {AB/A, A/AA, AA/B}
  2) {B/CA, A/AB, CA/A, ABC/C}
  3) {X/Y, A/B}
  4) {ACA/AC, AB/A, AB/B}

  The answer for 1) is (AB/A, AA/B, A/AA, A/AA), and for 2) is (A/AB, B/CA, CA/A, A/AB, ABC/C). The question of uniqueness of the solution is acknowledged in passing or investigated. For 3) no solution is possible since there are no common letters at the lower and upper halves. For 4) no solution is possible since all strings at the upper halves are longer than the strings at the lower halves such that the upper concatenation will always be longer than the lower concatenation and thus they can never coincide. Students are asked to share their experience as to how they came to know that the problem could not be solved. While the objective content of a judgment lies outside of feeling, for a student to be convinced of the correctness of a judgment then feeling must develop. Most of the Saratoga students had an experience of the knowledge coming to them as an emerging insight after considerable pondering; a few had continued to search for a solution.
- The intent for working with the soul capacities of a student on this day is as follows. First, a natural enthusiasm arises when working in a team. These feeling*s* enter and permeate her will that is then raised to activity and engaged by the manual activity of making the dominos, and then trying domino configurations to find a solution. Through the need to figure out the correspondence the student's will stirs her thinking. Thinking in turn enters her will to give intentionality to her nimble fingers, e.g., is the correspondence beginning to appear? Following the hands-in work, an imaginative recollection of the facts stirs her feelings in relation to the hands-in work. These feelings are the fundament needed for the next step.

Overview of content for Day Seven.

- The intent for this day is that, following sleep, pictures are present and available to the thinking. Reflections and discussions are possible that, through inspiration, bring to consciousness in the child the general rules that stand behind the observed examples from yesterday. This is the context within which the teacher presents the concept of undecidability. Will that can draw on the precedent of the hands-in work enters her thinking to form, direct, and connect these thoughts. By deferring until this day the conjoining of concept with the percepts from the previous day, a practice for “living thinking” is cultivated. The undecidability concept is introduced today as a plausibility argument – it could be replaced later in life by a formal mathematical proof while leaving intact the student’s underpinning experimental experience with dominoes.
- The students are asked to determine an algorithm that takes any array of domino types as input and produces as output a solution to the problem or asserts that no solution is possible. A lot of exploration of approaches is appropriate here. Ultimately the teacher might need to create a systematic path such as the following. First a solution for 1 domino – either the top string matches the bottom or it does not. Then a solution for 2 dominos - using the experience from the previous day the students could test whether all letters on upper halves are different from all letters at lower halves and whether each domino has a longer string at the upper half than at the lower half. However, this will not cover all cases. A proliferation of cases starts up as differing numbers of symbols are considered. The possibility arises that testing possible solutions could go on forever for lack of a comprehensive set of conditions for no solution. This insight is confirmed by the teacher to be true for the general procedure for any array of domino types – the sought for algorithm does not exist. Thus we cannot find an algorithm to solve this problem. Therefore we could not mechanize the solution with software. At the same time, people seem to be able to solve new particular instances of this undecidable problem. This opens up a deep truth in computing and a deep truth concerning the relationship between people and computing. They deserve some quiet space in which to be respectfully held.
- Students are given further examples of how the undecidability problem manifests in today's computers:
  1) It is not possible to construct an algorithm that can determine, for all procedures, whether another procedure will halt.
  2) It is not possible to construct an algorithm that proves, for all algorithms, that an algorithm is correct.
  3) It is not possible to construct an algorithm that, for all virus algorithms, will detect that virus. Thus a "virus shield" can never detect all possible viruses. Only once a new virus has been found a new checking condition can be added to the virus shield. The new "virus" has to be found and identified by a human!

Overview of content for Day Eight.

- In 1950 Alan Turing proposed a test: There are two keyboards in front of you, one connected to a computer, the other leads to a person. You type in questions on any topic you like; both the computer and the human type back responses that you read on the respective computer screens. If you cannot reliably determine which was the person and which the machine, then we say the machine has passed the Turing test. It is noted that the Loebner Prize Contest at http://ww.surrey.ac.uk/dwrc/loebner runs annually and awaits a winner that passes such an unrestricted Turing Test.
- However, we can restrict the test to give the computer a fighting chance. One such restricted test has been done where the problem area is restricted to chess games and the second restriction that you only see the recorded moves of a previous game. Now you must state if either opponent, or possibly both or possibly neither, was a computer. In an informal experiment, Garry Kasparov could occasionally, but not reliably, guess from recorded games whether opponents were human or machine. Note that many strategies would be open to you in a test without this second restriction, e.g., you might deliberately lose the game in a way that might reveal the identity of your opponent.
- The Turing test is set up in the classroom. The Eliza program is downloaded[16] or run off a web site.[17] Saratoga students did not run the Turing test for logistical reasons, rather they ran Eliza directly. Saratoga students considered certain behavior as telltale that Eliza is not human:
  - Some repetitiveness.
  - Some nonsense statements.
  - Sometimes combined responses to multiple statements in an unthinking awkward way.
  - Showed a lack of initiative.
  - Sometimes had blanks in responses as if the form of the response was predetermined but a keyword was missing.
  - Despite the significant issues above, Eliza corrected trivial typographical errors by the students.

  These characteristics were considered telltale even if Eliza were to be considered a Rogerian psychotherapist[18] who can legitimately communicate as if they had a limited knowledge of reality.
- "Improvements" on Eliza, such as ALICE[19] and Ella[20], have fewer telltale behaviors. Saratoga students felt that some repetitiveness and nonsense statement remained.
- The student's experience of Eliza, ALICE, and Ella is considered in terms of algorithms. What kind of algorithm might be at work, e.g., looking for keywords? How might the telltale behaviors be cleverly mitigated?

Overview of content for Day Nine.

- Reflecting on yesterday, people seem to be able to solve new particular instances of undecidable problems but algorithms, by their very character, cannot. This positions the class to enter into a general but concrete discussion of the relationship between computing and human intelligence. People, through free personal insight can think through problems for which no algorithm exists whereas computing machines are constrained to interpret algorithms. Further, people can then decide to act on their thinking in this way or that way whereas computing machines are constrained to "act" in a predetermined configuration. To broaden the discussion consider a social problem such as "how can I make my two friends speak to each other again". Each case is another case. It is fundamentally not a mathematical problem, e.g., there is no meaning for our friends in estimating a probability based on outcomes of "similar" cases even if abstractions were used to characterize a similar case.
- To give further concrete perspective to the capacities of computing machines, and the ingenuity of their creators, a way of thinking about a legitimate "superiority" of electronic computers is discussed. When children learn about animals in 4th Grade, the teacher introduces them by relating their capabilities and form to those of people. Specific animals can do specific things "better" than people but they are rather specialized in these talents. Current students are surrounded by "smart objects". Robots or programs can exhibit behavior that is in some sense superior to people, e.g., quickly, accurately, and tirelessly performing a repetitive task that can be expressed algorithmically. This is a narrow superiority that can be healthily granted to the machine. An aspect that makes the narrowness hard to see sometimes is that computers appear to be non-specialized because of their programmability – their functionality depends on the program they interpret. Nevertheless, the algorithmic thinking style is so limited that overall the capacity is narrow.

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[16] For example, http://www.spaceports.com/~sjlaven/eliza.htm has an MS-DOS program for download.
[17] For example, http://www-ai.ijs.si/eliza/eliza.html has a web-based version of Eliza.
[18] Weizenbaum, Joseph. "ELIZA – A Computer Program For the Study of Natural Language Communication Between Man and Machine". *Communications of the ACM*. January 1966.
[19] http://www.alicebot.org
[20] http://www.ellaz.com

- The historical mission of computing is explored. This can provide guiding context for a more detailed symptomatic treatment of the historical turning points in the relationship of people with computing. The development of technology affords us an opportunity for freedom or for bondage.[21] Consider digging a hole in the ground in three different ways:

Method used	Physical willpower engaged
Bare hands	Most
Spade	Some, e.g., drive spade into ground with foot
Back hoe	Least, i.e., move joystick to and fro

  The increasing physical cutting and leverage afforded by the machines increasingly separates physical willpower from the task at hand. Then willpower is freed up for other things, e.g., to be applied to creative thinking to better plan the project. However, there is a temptation to become dependent on the machines and if we forget to apply some of our increased thinking opportunity to learning how to fix back hoes then if the back hoe breaks we have to abandon the job (at least until the "expert" arrives). In recent times, the analogous situation for thinking has arisen. This is the contribution of the computer. Consider three ways to perform addition of a list of numbers:

Method used	Thinking engaged
Mentally	Most
Abacus	Some, e.g., individual calculations are done mentally
Modern personal computer	Least, e.g., operate the control panel on a spreadsheet

  The increasing intellectual cutting and leverage afforded by the machines increasingly separates thinking from the task in mind. Then thinking is freed up for other things, e.g., to be applied to creative thinking to better plan the project. We must remain mindful of potential downfalls such as the following:
  - Being carried away by a feeling of omnipotence. Viewing oneself as inheriting the power of the computer, to be wielded without the input of intellectual or physical work. That is an illusion of self-development.
  - Becoming disempowered with respect to the task in mind. This could happen in multiple ways, e.g., losing the meaning of the task, giving away authority over the task, becoming physically will-bound and mentally paralyzed as the provider of input to the machine, becoming dependent and living in fear of a computer break down.
  - To offload lesser thinking tasks but then not do the inner work of more advanced thinking to transcend to higher consciousness. This could lead to one's own thinking degrading to an algorithmic computing intelligence.
- This is all suitable for an essay question. Saratoga students variously tackled "Are computers becoming smarter than people or are people becoming dumber than computers" or "What does a healthy relationship with a computer look like".

[21] For an extensive discussion see Setzer, Valdemar W. "The Mission of Technology". 1996. Available at http://www.ime.usp.br/~vwsetzer.

3. 11^{th} grade: stored program control.

- Several previous activities prepare the way for an explicit treatment of stored program control in 11^{th} grade. In the 10^{th} grade curriculum, analogies are made to memory, processing, and data input/output in a modern electronic computer for each of the machines made. In the 9^{th} grade, the von Neumann architecture is identified in terms of the components of a PC.
- The next step is a detailed human emulation of the components of the von Neumann machine. Valdemar Setzer's Paper Computer exercise[22] has thoroughly worked this through. This gives a concrete first experience of an instruction set and the fetch-decode-execute cycle. The content is tested and ready for classroom use by others.
- The next step introduces machine language. Valdemar Setzer's machine language exercise[23], including an MS-DOS machine emulator available for download, has thoroughly worked this through. Using the emulator is straightforward and the programming gives a concrete and understandable experience of how the instruction set, machine language, and the fetch-decode-execute cycle all work. The content is tested and ready for classroom use by others.

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[22] Setzer, Valdemar W. "The "Paper Computer" - A Pedagogical Activity for the Introduction of Basic Concepts of Computers". 2000. Available at http://www.ime.usp.br/~vwsetzer.
[23] Setzer, Valdemar W. "The HIPO Computer - A Tool for Teaching Basic Computer Principles through Machine Language". 2000. Available at http://www.ime.usp.br/~vwsetzer.

4. 11th grade: algorithms.

A "day" here happens to be a 45 minute class – the 12th grade Saratoga students, for whom these 11th grade topics were included, had 4 such distinct classes per week.

Outline of lesson plans.

- Day One
  - History thread: Baron de Prony's human computing teams.
- Day Two
  - Computing machine thread: experiencing being a human computing team.
- Day Three
  - Algorithm thread: how does the method of differences algorithm work?
  - History thread: what happened to human computing teams?
- Day Four
  - History thread: Muhammad ibn-Musa Al-Khowarizmi.
  - Algorithm thread: the character of an algorithm.
- Day Five.
  - Algorithm thread: well-designed algorithms.
- Day Six
  - Algorithm thread: development of sorting algorithms.
- Day Seven
  - Algorithm thread: testing of sorting algorithms.
- Day Eight
  - Algorithm thread: analysis of sorting algorithms.

Overview of content for Day One.

- The teacher gives the story of Baron Gaspard de Prony's invention of computing teams in the 1790s[24]. He was commissioned by the French government to prepare extensive mathematical tables, e.g., logarithms and trigonometric functions of angles. Accurate tables were essential for correct navigation; sailor's lives were at stake, and for other purposes. Prony was inspired by the concept of division of labor put forth by Adam Smith in Wealth of Nations[25], recently published in 1776. The first chapter, on division of labor, describes how manufacturing processes can be broken down into small steps, each performed repetitively by specialized workers. Most of de Prony's workforce came from the ranks of unemployed hairdressers, the fashion for powdered wigs having died out with the French Revolution. Prony organized his force of mathematicians into three echelons: at the top were six outstanding men that did the analytical work to determine what was to be computed by which formula; the second echelon was comprised of eight skilled calculators who knew algebra and could use the formulas to make detailed calculations of values at regular intervals and carry the work to where it became the repetitive addition or subtraction of numbers in difference tables. Then the third echelon, comprised of seventy men, did only addition and subtraction rapidly and fairly accurately. To increase accuracy, each number was computed at least twice in different parts of France to prevent collaboration between the two groups. The final tables still contained errors due to both calculation and the process of setting the table into type.
- The teacher describes the need for de Prony's team to efficiently evaluate functions such as sin(x) = x – 1/3!x_ + … for x=0°, 0.1°, 0.2°, … 90°. Then a calculation without fractions is tackled for a practical first experience. Consider the evaluation of 41+x+x_+x_ for x=0,1,2…50. First, each student calculates the whole expression individually and reports the result to be put on the board. Saratoga student results were accurate through x=5; the time required became longer as x increased, taking around a minute for x=5; for x=6 we saw our first errors – the complexity had reached a threshold. The students formulate some estimate of the long time required to individually evaluate the expression through x=100 and the likely accuracy of the results.
- Adam Smith's description of division of labor is read. Saratoga students found the individual choices and sacrifices necessary to increased productivity by specialization pertinent to choices they face in their near future beyond high

[24] Williams, Michael. *A History of Computing Technology.* Los Alamitos: IEEE Computer Society Press, 1997.
[25] Smith, Adam. *An Inquiry Into the Nature and Causes of the Wealth of Nations.* Available from The Adam Smith Institute at http://www.adamsmith.org/smith/won-intro.htm.

school. A student question of "Can the division of labor be applied to improve productivity of our calculations?" arose out of this experience and we slept on it.

- The intent for this day is that the history content stirs in the student her feelings of enthusiasm or love for people and their relationship with computers, e.g., caring for the lives of sailors which depend on accurate tables, or empathizing with de Prony and his formidable challenge. These feelings enter and permeate her will that is then raised to activity and engaged by the calculations, which are done manually using paper and pencil. Through the experienced need to improve productivity her will stirs her thinking. An imaginative recollection of the facts stir her feelings in relation to the question of how could a team of people, working in the manner described by Adam Smith, do better? These feelings are the fundament needed for the next step.

Overview of content for Day Two.

- The intent for this day is that following sleep, pictures are present and available to the thinking concerning a manufacturing assembly line of people. The next activity fulfills this imaginative rehearsal.
- The students form teams of 3 people each. Each team chooses a city of France for their location. The teacher, playing the second echelon, assigns job descriptions and a work flow to each member of each team, starting the computation at x=3 to avoid job-complicating initial conditions that don't require all three team members:
  - Person 3 must write the value of x on a slip of paper, add 6 to their last result (using 8 for x=3) and pass the slip to person 2.
  - Person 2 must add this input to their previous total (using 11 for x=3) and pass the slip to person 1.
  - Person 1 must add this input to their previous total (using 55 for x=3); if a comparison of the result with the checkpoint sheet shows a discrepancy then the process must be halted and a resolution found, else the result is stacked up.
  - An accuracy check is performed for x=3,4,5,6,7,8 from earlier calculations by individuals. The teacher as second echelon issues the checkpoint sheet showing the values of the function for x=10,20,30,40,50. Checking discrepancies can be a significant task because a problem at the checkpoint for x=20, say, could have arisen at x=19,18,17,16,15,14,13,12, or 11. Each team evaluates the function for x=3,4,5, ….. 50.
- Saratoga students faster teams reached x=50 comfortably within the 45 minute period. A few teams became bogged down with discrepancies. The students made the following observations:
  - High speed of calculation compared to individual effort.
  - Low skill requirement (just adding) per third echelon individual.
  - Opaqueness of the underlying method to the third echelon individuals; only the second echelon and first echelon know what is going on. The accuracy achieved by the team was experienced as magical, with strong power from an unknown source. A student question of "How does it work?" arose out of this experience and we slept on it.
- The intent for this day is that the will is first stirred by being entered and permeated with feelings of enthusiasm fostered by working in a team as well as a little light-hearted competitiveness with other teams. The will that is then raised to activity is engaged by active doing with the hands. In turn, through the need to compute accurately the student's will stirs her thinking. Thinking in turn enters her will to give intentionality to the calculations with paper and pencil by her nimble fingers. Following the hands-in work, an imaginative recollection of the facts stirs her feelings in relation to the hands-in work. These feelings are the fundament needed for the next step.

Overview of content for Day Three.

- The intent for this day is that following sleep, pictures are present and available to the thinking. Reflections and discussions are possible that, through inspiration, bring to consciousness in the child the rules that stand behind the observed facts. This is the context within which the teacher presents the method of differences algorithm. Will, which can draw on the precedent of the hands-in work, enters her thinking to form, direct, and connect these thoughts. By deferring until this day the conjoining of concept with the percepts from the previous day, a practice for "living thinking" is cultivated.
- The class sets out to answer the question of "how does the computation work". Saratoga students noted that all the multiplications in 41+x+x_+x_ had been mysteriously converted to additions, but nobody could see into the source of the magical power. The teacher explains and names "the method of differences" using a table such as below:

x	41+x+x_+x_	1st difference	2nd difference	3rd difference
0	41	empty	empty	empty
1	44	3	empty	empty

2	55	11	8	empty
3	80	25	14	6
4	125	45	20	6
5	196	71	26	6

♦ Then the table is shown to “run backwards” to effect the computation:

x	person 1	person 2	person 3	3rd difference
2	55	+ 11	+ 8	+ empty
3	= 80	= 25	=14	6

♦ Saratoga students were already interested in methods of mechanizing the tedium. They identified with the intention behind Charles Babbage’s letter to Sir Humphrey Davy, president of the Royal Society, July 3rd, 1822, referring to the third echelon of de Prony’s calculating teams:

> “The intolerable labour and fatiguing monotony of a continued repetition of similar arithmetical calculations, first excited the desire, and afterwards suggested the idea of a machine, which, by the aid of gravity or any other moving power, should become a substitute for one of the lowest operations of the human intellect.”

A reference to the local consumer electronics store “Babbages” was interesting to Saratoga students.

♦ The historical perspective is given that human computing teams persisted until the 1940s. For example, during the early 1940's, Kay McNulty, a recent math graduate from Chestnut Hill College, was employed along with about 75 other young female mathematicians as a "computer" by the University of Pennsylvania's Moore School of Engineering

Figure: Kay McNulty worked as a computer.

These "computers" were responsible for making calculations for tables of firing and bombing trajectories, as part of the war effort. This exemplifies the oldest dictionary definition of computer as “one who computes”, predating references to electronic and mechanical machines. The need to perform the calculations more quickly prompted the development of the ENIAC, the world's first electronic digital computer, in 1946. Kay McNulty Mauchly Antonelli recalls[26] computing in 1946:

> We did have desk calculators at that time, mechanical and driven with electric motors, that could do simple arithmetic. You'd do a multiplication and when the answer appeared, you had to write it down to reenter it into the machine to do the next calculation. We were preparing a firing table for each gun, with maybe 1,800 simple trajectories. To hand-compute just one of these trajectories took 30 or 40 hours of sitting at a desk with paper and a calculator. As you can imagine, they were soon running out of young women to do the calculations. Actually, my title working for the ballistics project was `computer.' The idea was that I not only did arithmetic but also made the decision on what to do next. ENIAC made me, one of the first `computers,' obsolete.

This use of the word computer, for human computer, persisted until the 1950’s at which time computing tasks were mechanized and people were redeployed as the first “programmers”.

♦ The method of differences is said to be an example of a well defined procedure called an “algorithm”, something to be followed up next lesson.

Overview of content for Day Four.

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[26] Strauss, Robert Strauss. *When Computers Were Born.* Times Mirror Company, 1996.

- The work of Muhammad ibn-Musa Al-Khowarizmi, 9th century Persian mathematician is reviewed. Given the recent US war with Iraq, facts concerning his teaching work at the Mathematical Institute in Baghdad and our heritage from his work carry a certain poignancy. The origin of the word "algorithm", from the Latin translation of his name as Algorismus, and the origin of the word "algebra", from his book "Kitab al jabr w'al muquabala", are explained.
- A characterization of an algorithm and a form are given, e.g., an algorithm is viewed as a sequence of unambiguous and mathematically defined steps that lead to a result and halts. Its form is thought of as:

  Step 1: Do X.
  Step 2: Do Y.
  Step 3: Do Z.
  Step 4: Halt.

  It is noted that the human computing team operated in this manner, with a well defined halt condition.
- A second example is the grade 2 algorithm for the sequence of steps for vertical addition with carrying, also called the ripple-carry addition algorithm. The students write down the calculation for 56+19=85, first the usual succinct way and then step by step something like the following (styles of presentation vary):

  Step 1: Write down the two numbers. So we write
  $$\begin{array}{r} 56 \\ \underline{+\ 29} \end{array}$$
  Step 2: Add the rightmost digits, i.e., 6+9=15.
  Step 3: Write down the 5. So we write
  $$\begin{array}{r} 56 \\ \underline{+\ 29} \\ 5 \end{array}$$
  Step 4: Carry the 1 to the second column. So we write
  $$\begin{array}{r} 1 \\ 56 \\ \underline{+\ 29} \\ 5 \end{array}$$
  Step 5: Add the carry-in 1 and the two digits in the second place from the right, i.e., 1+5+2=8.
  Step 7: Write down the 8. So we write
  $$\begin{array}{r} 1 \\ 56 \\ \underline{+\ 29} \\ 85 \end{array}$$
  Step 8: Halt.

  In the context of this familiar and transparent algorithm, it is noted that that there is no permissiveness, no room for creativity. Every addition is to be done the same way. The lively thought process of the algorithm designer has come to rest. Computation proceeds with the interpretation of the finished thought embodied in the algorithm. Such is the character of an algorithm. This second grade algorithm also illustrates the tradeoff compared to 1st grade counting that 2nd grade vertical addition is faster but less transparent as to how it works.
- To enter further into the characterization above we can ask if all procedures are algorithms. Consider the directions for making cheese sauce[27]:

  Step 1: Melt in saucepan 2 tablespoons butter
  Step 2: Stir in until blended 2 tablespoons flour
  Step 3: Stir in slowly 1_ cups milk
  Step 4: When the sauce is smooth and boiling reduce the heat and stir in: 1 cup or less mild grated cheese or diced processed cheese
  Step 5: Season the sauce with _ teaspoon salt, 1/8 teaspoon paprika, a few grains cayenne, (1/2 teaspoon dry mustard)
  Step 6: Stir the sauce until the cheese is melted.

  Using the characterization above: sequence of steps – yes; unambiguous steps – no; mathematically defined steps – no; lead to a result – yes; halts - yes. Overall, not an algorithm. Other examples such as changing a car tire are used to bring out the exacting narrowness and high degree of will in thinking required to create an algorithm.
- With the distinctive character of algorithms in mind we can consider one possible characterization of "computing" as the development and analysis of algorithms and means for interpreting them.[28]

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[27] Rombauer, Irma S. and Becker, Marion Rombauer. *The Joy of Cooking.* The Bobbs-Merrill Company, Inc., 1953. Page 430.

- The intent for this day is that the history content stirs in the student her feelings of admiration or appreciation for Muhammad's achievements. These feelings enter and permeate her will that is then raised to activity and engaged by the vertical addition calculation. Activity of limbs further stirs the will. Through the need to distinguish the steps in the addition her will stirs her thinking. Thinking in turn enters her will to give intentionality to her nimble fingers in the physical layout of the steps. Following the hands-in work, an imaginative recollection of the facts will stir her feelings in relation to the hands-in work. These feelings are the fundament needed for the next step.

Overview of content for Day Five.

- The intent for this day is that following sleep, pictures will be present and available to the thinking. Reflections and discussions are possible that, through inspiration, bring to consciousness in the child the general rules that stand behind the observed examples of algorithms. This is the context within which the teacher presents the concept of a well-designed algorithm. Will, which can draw on the precedent of the hands-in work, enters her thinking to form, direct, and connect these thoughts. By deferring until this day the conjoining of concept with the percepts from the previous day, a practice for "living thinking" is cultivated.
- So, reflecting on the method of differences and the vertical addition algorithm we ask what we should look for in a "well designed" algorithm. Computer scientists, programmers, and systems analysts are concerned with the development of well-designed algorithms. It must have the characteristic attributes as above. What other attributes would be desirable? This is likened to evaluating a car, e.g., it must have an engine, and it is desirable to be fuel-efficient. Students attributes for a well-designed algorithm are listed:
  - Must-have attributes:
    1. Sequence of steps. (Specific order.)
    2. Unambiguous steps. (Unlike seasoning with a "few" grains of cayenne.)
    3. Mathematically defined steps. (Prerequisite to mechanization.)
    4. Lead to a result. (Definite outcome.)
    5. Halts. (E.g., a procedure for calculating pi exactly would not.)
  - Desirable attributes:
    6. Suggestive name.
    7. Correctness. (The outcome is correct for the problem being solved.)
    8. General. (E.g., addition works for any two numbers.)
    9. Easy to understand. (For maintenance and programming.)
    10. Efficiency. (Fewest steps => minimum time taken to compute measured as a function of the size of its input. Often, there is a tradeoff: maximum speed is sacrificed for greater ease of understanding or vice versa.)
- Efficiency seems like an important attribute, e.g., if the compute time were to exceed one's lifetime that could be disappointing. So we turn to the study of algorithm efficiency.

Overview of content for Day Six.

- This exercise draws heavily on the work of V.W. Setzer and F.H. Carvalheiro.[29]
- A motivation for sorting algorithms is developed. For example, Saratoga students used a start up business scenario. Suppose we want to provide a competitive directory assistance service in New York. A profitable differentiating service is projected to be fast reverse look up by telephone number to find a person's name and address. From a supplier we could buy 20,000,000 records ordered alphabetically by personal and business names. We would need to sort them by order of telephone number to yield a list to be used for our fast reverse lookups. How long would it take to sort the 20,000,000 records? Suppose we wanted then to expand to a nationwide service using multiple 100,000s of records?
- Students organize into pairs for this exercise, both for a warming social interaction and for a helpful separation of concerns in the task. Each pair creates 8 slips of paper from one writing sheet. On each of 8 slips of paper one partner writes a counting number. Exactly 8 numbers is not essential. However, a small number is chosen so the units of work had manageable complexity and a power of 2 simplifies later discussion of binary merge sort. Then the other partner places them one at a time in a row, counting numbers face down, in a random order

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[28] I am using "computing" as a synonym for "computer science". I am not using "computer science" because the word suggests formalism that does not belong in high school.
[29] Setzer, V.W. and Carvalheiro, F.H. "Algorithms and Their Analysis." 2000. Available at http://www.ime.usp.br/~vwsetzer.
Setzer, V.W. "New Chapter 5 on Algorithms." 2002. Available at http://www.ime.usp.br/~vwsetzer.

while the writer does not look. Now neither partner knows the order of the 8 numbers. The task is set by the teacher as finding, writing down, and naming an approach to rearranging the slips in order of value increasing from left to right subject to the following constraints:

- You can look at the number on at most 2 slips at a time.
- You must forget the contents of a slip after it is placed face down.
- You can compare 2 lifted strips, determining which contains the smaller number.
- You can exchange two strips in position.

While the focus is on algorithm development independent of computing machine, a brief justification for these otherwise arbitrary constraints is given by reference to electronic computer memory locations and processor functions, e.g., they derive from the form of CPU instruction for comparing or exchanging memory location contents that can reference just one memory location in addition to the accumulator.

- The students are asked to do multiple runs to satisfy themselves that their algorithm works for any configuration of their 8 numbers. Then each pair of students is asked to conduct a sterner test by handing over their written algorithm to another pair to see if the other pair can successfully interpret the algorithm without any subsequent help from the authors. This gives an opportunity for the social experience of teamwork where one's peers are objectively evaluating one's work. Refinements of each write up are made so as to satisfy the must-have attributes of an algorithm. The Saratoga students found significant challenge in the sharp focus and perseverance needed to work through to an algorithm.
- The intent for working with the soul capacities of a student for this day is as follows. Through the warmth of teamwork and by active doing with their hands the will is stirred. Through the need to figure out how to order the numbers the student's will stirs her thinking. Thinking in turn enters her will to give intentionality to her nimble fingers, e.g., are the numbers actually becoming sorted?

Overview of content for Day Seven.

- The mutual testing is completed. Saratoga students were impressed by the fact that testing and refinement took as long as the original development of something that seemed to work. The students thought that testing by other people was a sound discipline but even that was not proof of correctness. However, the prospect of systematically checking every possible configuration of numbers seemed daunting.
- Following testing, the students count the number of comparisons and exchanges for two runs: for the slips in random order and for the slips to be in exactly the reverse order. The measurement of comparisons and exchanges is completed and tabulated. How to scale up to a "large" number, say 20 million numbers, becomes a student question. A general way to quantify the operations is required.
- The intent for working with the soul capacities of a student for this day is as follows. By active doing with their hands the limb activity stirs the will. Through the need to figure out how to order the numbers the student's will stirs her thinking. Thinking in turn enters her will to give intentionality to her nimble fingers, e.g., are the numbers becoming sorted? Following the hands-in work, an imaginative recollection of the facts stirs her feelings in relation to the hands-in work. These feelings are the fundament needed for the next step.

Overview of content for Day Eight.

- The intent for this day is that, following sleep, pictures are present and available to the thinking. Reflections and discussions are possible that, through inspiration, bring to consciousness in the child the algorithmic rules that stand behind the observed facts from the first two days. This is the context within which the teacher presents the concepts of algorithm analysis. Will, which can draw on the precedent of the hands-in work, enters her thinking to form, direct, and connect these thoughts. By deferring until this day the conjoining of concept with the percepts from the previous two days, a practice for "living thinking" is cultivated.
- At this stage, either the student's actual algorithms are analyzed or "well-known" algorithms that match or are close to the student's algorithms are introduced to simplify the analysis. This choice would seem to be highly dependent on the level of interest, will, and algebraic ability of the students. Saratoga students created algorithms that were bubble sort, selection sort, a mix of bubble sort and selection sort, or a mix of binary merge sort and bubble sort. Assuming the choice is to work with "well-known" algorithms, the teacher proceeds to a calculation of operations. Hybrid algorithms are analyzed by students depending on their level of interest. In the manner described by Setzer et al[30] the well-known algorithms are analyzed for their number of comparisons and exchanges for a variable number N of items. For N=8 the results are compared with the student counts - they should be comparable. For large N, such as 20M, the order of magnitude comparisons and exchanges are found, e.g., ~10**14 for 20M comparisons needed by bubble sort, ~10**9 for 20M comparisons for binary merge sort. Such order of magnitude calculations call upon feeling in thinking.
- A lower bound for compute time is found with a method such as:
  CPU execution time = clock cycles/instruction x seconds/clock cycle ~ 10 x 1/1Ghz, say ~ 10**-8 s/instruction
  So for ~10**14 instructions we have ~ 20 days and for ~10**9 instructions we have ~5 seconds.
- For an arbitrary large N, order of magnitude results are derived such as bubble sort and selection sort comparisons = O(N_) and binary merge sort comparisons = $O(N\log_2 N)$. The Saratoga students used the ration for N=200M records to N=20M records. This showed that the bubble sort and selection sort compute times were impractical and the binary merge sort is necessary.
- The next step is announced: to express our algorithm in a programming language for insertion into and interpretation by a computing machine. Reflection on the significant intellectual effort expended so far clarifies the distinction between <u>what</u> the computing machine is instructed to do distinct from <u>how</u> it is so instructed. The algorithm is seen as the bearer of the person's intention of the task and this highly creative work has now come to rest. Subsequent steps must, as faithfully as possible subject to machine limitations, carry out this intention.
- Programming is now introduced. This activity is positioned as already the third of a sequence of software development lifecycle steps: problem definition -> algorithm development and analysis -> programming. The role of the programming language is only a syntactical role – the semantic content is pre-determined by the algorithm. Saratoga students used the Microsoft Q Basic language. Although a programming lesson is not further outlined here, it is recommended that students experience socially oriented parts of the software process, e.g., test or upgrade each other's programs, to balance the soul experience.

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[30] Setzer, V.W. and Carvalheiro, F.H. "Algorithms and Their Analysis." 2000. Available at http://www.ime.usp.br/~vwsetzer.
Setzer, V.W. "New Chapter 5 on Algorithms." 2002. Available at http://www.ime.usp.br/~vwsetzer.

5. 10^{th} grade: how have computing machines and their relationship with people evolved?

A "day" here happens to be a double 45 minute class – the 10th grade Saratoga students had 2 such distinct classes per week. Double periods worked well for this hands-in course, allowing sufficient time to set out the materials and tools, time to do the work, and time to put away. Twice per week worked well in terms of a rhythm of working with sleep across the two days of each week. Each topic below except the transistor circuits was confined to one week.

Outline of lesson plans.

- Day One
  - History thread: mental math, finger math, and motivation for the abacus
  - Computing machine thread: build an abacus from clay.
- Day Two
  - Computing machine thread: operate the abacus.
  - Computing machine thread: find limitations of the abacus.
- Day Three
  - History thread: John Napier
  - Computing machine thread: build Napier's rods from wood.
- Day Four
  - Computing machine thread: operate the rods for multiplication and division.
  - Computing machine thread: find limitations of the rods.
- Day Five
  - Computing machine thread: operate the rods for square root and cube root.
- Day Six
  - History thread: Baron de Prony's human computing teams.
  - Computing machine thread: experiencing being a human computing team.
  - History thread: what happened to human computing teams?
- Day Seven
  - History thread: Henry Morse and the first telegraph
  - Computing machine thread: build and operate a telegraph circuit.
- Day Eight
  - History thread: George Boole.
  - Computing machine thread: build AND, OR, NOT, NAND, NOR gates from DC components.
- Day Nine
  - History thread: Bardeen, Brittain, Shockley and the transistor.
  - Computing machine thread: build AND, OR, NOT gates from transistors.
- Day Ten
  - Computing machine thread: introduction to binary arithmetic
  - Computing machine thread: combine gates into NOR gates and then into half adder circuits.
- Day Eleven
  - Computing machine thread: combine circuits into a 2-bit adder.
- Day Twelve
  - Computing machine thread: electronic calculators and their limitations.
- Day Thirteen
  - Computing machine thread: electronic calculators and their limitations.
  - Analysis of the trends in how people relate to computing machines throughout history

Overview of content for Day One.

- Begin with an imagination of the paperless world of ancient Egypt, where only a privileged few had papyrus, and before. Math could be done mentally. Do a few exercises to show how calculations soon become difficult, e.g., 989+989 might be of interest for a commercial transaction but is difficult to do mentally. How can we find help? Fingers were a first step to externalizing the arithmetic. Show various representation schemes, e.g., one hand can show up to 9 or up to 99 depending on the scheme.[31] Show Roman counting. Unary at first. Then something special happens at 5 that is resonant with the human hand – V is used, based on the thumb opposed to the four collected fingers. Then something special happens at 10 that is resonant with the human hand – X is used, consisting of two V's. In this way, a 10's representational system is based on 2 hands. A 20's counting system, evidence of which remains in the French vingt, quatre vingt, quatre vingt dix, includes the toes. This relationship to the form of the human being is referred to for each computing machine design until we arrive at binary systems where a non-human basis is used.
- How can we free up our fingers to do other things while the computation is proceeding? Enter the abacus. The students work in pairs for a warming teamwork connection to build a clay tablet and 81 counters. One partner cuts a _" slice from a 6"x6" block of clay, rolls it to a _"x7_"x7_" tablet and scores it with 9 parallel grooves. The teacher cuts one or more slices for the counters, rolls to a _" sheet and divides among the other partners to cut 81 clay cubes per pair for counters.
- The intent for working with the soul capacities of a student for this day is as follows. The will is first stirred with active manual work for the finger math. Teamwork for the clay work creates enthusiasm that, together with associated limb activity, further stirs the will. Through the need to figure out the correct construction of the clay tablet and counters the student's will stirs her thinking. Thinking in turn enters her will to give intentionality to her nimble fingers to form the clay. Following the hands-in work, an imaginative recollection of the facts stirs her feelings in relation to the hands-in work. These feelings are the fundament needed for the next step.

Overview of content for Day Two.

- The intent for this day is that, following sleep, pictures are present and available to the thinking. Reflections and discussions are possible that, through inspiration, bring to consciousness in the child the rules that stand behind the observed form from yesterday. This is the context within which the teacher presents the concepts of computing using the abacus. Will, which can draw on the precedent of the hands-in work, enters her thinking to form, direct, and connect these thoughts. By deferring until this day the conjoining of concept with the percepts from the previous day, a practice for "living thinking" is cultivated.
- The teacher shows how to represent numbers on the abacus, with up to 9 counters per groove. The largest number representable per tablet is 999,999,999; the smallest number is 0. The word capacity is introduced for this range. Later exercises can introduce a decimal point, marked with an x in the clay to fix it, and change the capacity.
- The sum 989+989 is redone on the abacus. There are so many counters that mistakes tend to occur. A "second generation" of abacus is created by representing 5 in a special position, namely between grooves - the form of the human hand is being played out. We redo 989+989 to confirm the "upgrade" is working. Students make observations, e.g., faster to manipulate the counters and faster to read the result.
- Now the number of counters has been reduced, the tablet can be made physically smaller. A "third generation", smaller, tablet is created by cutting the still soft tablet in half – each student now has one. The counters are shared out. Already we have experience a trend to pervasive, smaller, less expensive, and faster computers.
- Do many calculations. There are a variety of algorithms (there is no need to use this word) for the arithmetic.[32]
- Is the smaller tablet "computationally as large"? The capacity is found to be the same. Physical size and computational size are separable. How could the capacity be changed? Do 500,000 x 2 that leads to overflow. The end result of moving the counters for this computation is to register 0. (Some electronic calculators do this.) To register the 1 in 1,000,000, we could add a finger, a 10th groove, or place two tablets side by side for 18 grooves to create an enormous expansion in capacity. Students can calculate how large is the expansion in capacity for just doubling the physical size. This is the 4th generation machine.
- What is the essence of the function that the abacus performs for the person? Saratoga students came to the opinion that it is a memory aid, storing interim and final results – the person is doing all the computation. When compared with the activity of mental arithmetic, this machine relieves the thinking and the fingers of any burden of memorizing results.
- Analogies are made to memory, processing, and data input/output in a modern electronic computer - this is preparing the way for an explicit treatment of stored program control in 11th grade.

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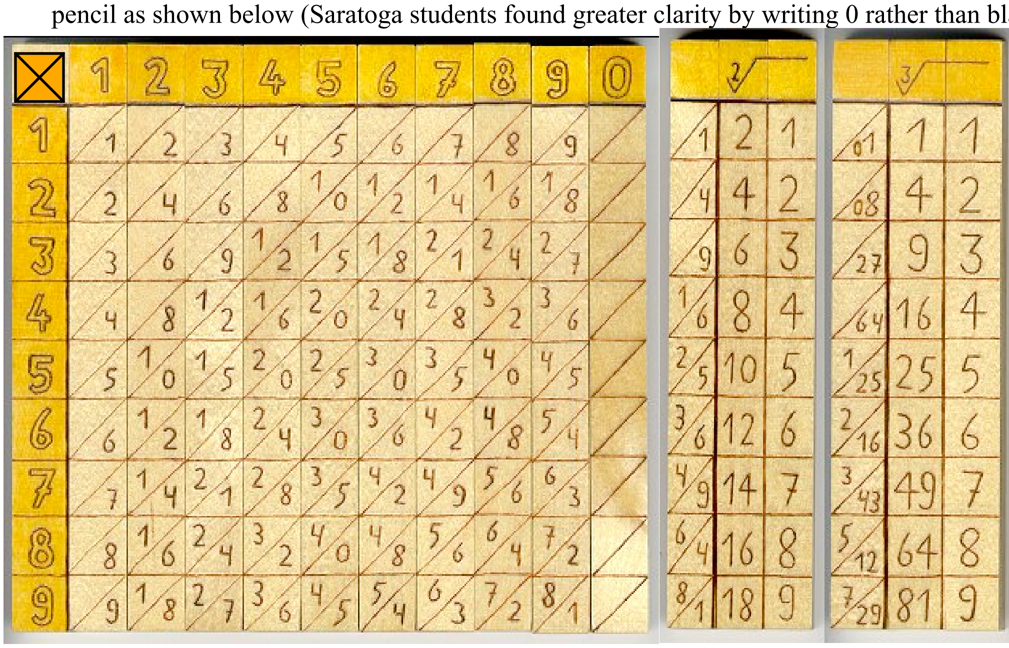
[31] Williams, Michael. *A History of Computing Technology.* Los Alamitos: IEEE Computer Society Press, 1997.
[32] Dilson, Jesse. *The Abacus: A pocket computer*. New York: St. Martin's Press, 1994.

- ♦ Transparency of operation (how easily can an individual understand the operation), power (speed and scope) of operation, limitations of operation (capacity and precision), and extent of human delegation (e.g., memory, processing, ….) are summarized.
- ♦ There are no surviving clay abaci. Various designs in other materials are considered, e.g., Greek marble table, metal Roman abacus, and wooden Chinese and Japanese abaci. The teacher describes how abaci are used today in businesses and schools in China and Japan.

Overview of content for Day Three.

- ♦ John Napier, born in 1550, was a Scottish baron who did math as a hobby. In the late 16th century advances in astronomy were hampered by laborious calculations. Napier took this up as his concern: “The difficulty and prolixity of calculation, the weariness of which is so apt to deter from the study of mathematics, I have always, with what little powers and little genius I possess, laboured to eradicate.”[33] His influential book Rabdologia (from the Greek words “rhabodos” and “logos” for “stick reasoning”) was published in 1617 shortly after his death.
- ♦ Each student cuts from a strip of wood a set of 13 rods, e.g., 11 rods _”x5” and 2 rods 1_”x5”, and marks them in pencil as shown below (Saratoga students found greater clarity by writing 0 rather than blanks):



- ♦ The intent for working with the soul capacities of a student for this day is as follows. The history content stirs in the student her feelings of enthusiasm for John Napier and his intention for reducing the burden of computation for others. These feelings enter and permeate her will that is then raised to activity and engaged by active doing with her hands to cut and mark the rods; the limb activity further stirs the will. Through the need to figure out the correct way of cutting and marking the rods, which is somewhat subtle for a 10th grader, the student’s will stirs her thinking. Thinking in turn enters her will to give intentionality to her nimble fingers.

[33] A translation by Mark Napier, a descendant of John, of a Latin letter of dedication to Alexander Seton appearing in the introduction to Napier’s *Rabdologia.*

Overview of content for Day Four.

- A graduated set of multiplications is done, e.g., 7x4, 71x4, 713x4, 7136x4, … and 713,652,408x4. What is the essence of the function that the rods perform for the person? The machine evidently embodies the multiplication tables to enable the conversion of x to +, provided one number is single digit.
- A graduated set of multiplications for multiple digits is done, requiring the person's memory, or paper and pencil, as an aid.
- A graduated set of divisions is done, using the rods "backwards".
- A decimal point is introduced into one of the numbers, e.g., 7.1x4, 71.3x4, 71.36x4.
- As for the abacus before, the capacity is determined. The unnecessary restriction of one rod per digit is acknowledged. The capacity is seen to be 9999….999 according to the number of 9 rods available; the machine is readily extensible such that the rate of growth of computational capacity far exceeds the rate of growth of physical size.
- The property that the machine does not overflow is identified, since the largest number to be registered, e.g., 999,999,999x9, is accommodated within the cells.
- The $\sqrt{\ }$ and $\sqrt[3]{\ }$ rods are operated[34]. The answers for simple cases are correct. The Saratoga students asked "How do they work?" and we slept on the question.
- The intent for working with the soul capacities of a student for this day is as follows. Will is raised to activity and engaged by active doing with her hands to manipulate the rods. Through the need to figure out the properties of the rods, the student's will stirs her thinking. Thinking in turn enters her will to give intentionality to her nimble fingers.

Overview of content for Day Five.

- The intent for this day is that, following sleep, pictures are present and available to the thinking. Reflections and discussions are possible that, through inspiration, bring to consciousness in the child the rules that stand behind the observed form from yesterday. This is the context within which the teacher presents the concepts of how to calculate square roots using Napier's rods. Will, which can draw on the precedent of the hands-in work, enters her thinking to form, direct, and connect these thoughts. By deferring until this day the conjoining of concept with the percepts from the previous day, a practice for "living thinking" is cultivated.
- An explanation of the process underlying the operation of the $\sqrt{\ }$ and $\sqrt[3]{\ }$ rods is given. More difficult problems are solved.
- The rods are raced against the abacus for fun and to investigate relative power.
- Analogies are made to memory, processing, and data input/output in a modern electronic computer - this is preparing the way for an explicit treatment of stored program control in 11th grade.
- Transparency of operation (how easily can an individual understand the operation), power (speed and scope) of operation, limitations of operation (capacity and precision), and extent of human delegation (e.g., memory, processing, ….) are summarized.

Overview of content for Day Six.

- The treatment of human computing teams is the same as for the 11th grade history thread and computing machine thread for that topic in Section 4 above. The underlying algorithm is deferred to grade 11.
- Saratoga students were highly enthusiastic about this activity. They were highly impressed by the productivity gain over individual effort due to division of labor.
- Transparency of operation (how easily can an individual understand the operation), power (speed and scope) of operation, limitations of operation (capacity and precision), and extent of human delegation (e.g., memory, processing, ….) are summarized.

Overview of content for Day Seven.

- The teacher gives the story of Samuel Morse and the electric telegraph. He was also a painter and sculptor. Saratoga students enjoyed examples of people with broad interests and skills – each of us could find an affinity with such a person. A focus is brought to the transition from a 10 state system, fashioned after the human hand, to a 2 state system characteristic of electrical circuits.

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[34] Some worked examples are available at http://www.qnet.fi/abehr/Achim/Calculators_Napier_rods2.html.
A complete explanation of the algorithm is given in Schuberth, Ernst. *Introduction of Advanced Arithmetical Operations for Waldorf School 7th Grades. The Calculation of a Square Root and Higher Roots.* Fair Oaks: AWSNA, 1999.

- The teacher builds one circuit, placing a battery and wires with alligator clips that is used to close the circuit in the remotest point in the classroom. Long wires run to a buzzer among the student body. This circuit is shown to be analogous to the telegraph circuit used to send Samuel Morse's first message from Washington D.C. to Baltimore in 1844. The single abstraction of a schematic is handed out and discussed.
- The teacher hands out a Morse Code alphabet and demonstrates signaling. Signaling is done by opening and closing the contact between two alligator clips. It is pointed out that printed dots and dashes preceded audible long and short sounds. To train the students to translate from sound, the teacher signals SOS (no spaces between the letters so does it really stand for Save Our Souls?) and Morse's first message "What hath God wrought." The students take turns signaling to the others students who write down the message. Saratoga students enjoyed this social experience.
- It is examined closely how long strings of dots and dashes arise from the need to use 2 symbols to differentiate 26 letters, 10 digits, and various symbols of punctuation.

Overview of content for Day Eight.

- The teacher tells the story of George Boole and Claude Shannon. Their diverse individual talents provide interest. In 1854, Boole published laws of logic in the form of an algebra.[35] We remember him whenever we use "AND, OR, and NOT" to connect search terms to find information in search engines. Boolean algebra was largely unknown and unused until 1938 when Claude Shannon published an article based on his master's thesis at MIT that showed how Boolean functions can be used to represent the functions of switches in electronic circuits. Shannon had provided electronics engineers with the mathematical tool they needed to design digital electronic circuits, as the students are soon to do.
- The students are organized into pairs for a warming social component and to help each other think through making circuits. The teacher explains the first exercise is to build a circuit to be used to manually turn a bulb on and off, and demonstrates the circuit. There is no intention here to volunteer how the components work – the 11th grade electricity and magnetism block could do that – just to demonstrate the behavior phenomenologically. The teacher hands out a physical diagram for the circuit with a component list (e.g., 1 double battery holder, 2 AAA batteries, 1 bulb holder, 1 bulb, 1 5 ohm resistor, 4 jumper leads). The student pairs pick up their components and make the circuit work as was explained and demonstrated. The class gathers in attention to be sure everyone thoroughly understands what they have done. A schematic for the circuit is handed out and the interpretation and usefulness of this abstraction discussed.
- The teacher explains the AND function, hands out a physical diagram for the circuit with a component list (1 more jumper lead). The student pairs pick up their components and figure out how to make the circuit work with this "and" condition. The class gathers in attention to be sure everyone thoroughly understands what they have done. A schematic is handed out and discussed.
- The teacher states the intention to do arithmetic with a combination of logic gates, by achieving the needed behavior. The students are asked to be patient to wait see this happen. To be able to express circuit behavior we need a vocabulary – the table below is handed out, and its contents compared with the observed And circuit behavior:

Switch A closed	**Switch B closed**	**Light on**
True	True	True
True	False	False
False	True	False
False	False	False

  The term "truth table" is introduced.
- In a similar manner, OR, NOT, NAND, NOR gates are built, physical diagrams, schematics, and truth tables studied. The students make at least one of these circuits from the teacher's verbal description, and not rely on handouts, so as to further internalize the disciplines.
- The exercises on this day introduce much new content for the students but with familiar physical components. This new content can now be carried into work with transistors that are a new and impenetrable technology for students this age.

Overview of content for Day Nine.

- The teacher gives some history of the invention of the transistor in 1947 by Bardeen, Brattain, and Shockley at Bell Laboratories in New Jersey.

[35] Boole, George. " An Investigation Into the Laws of Thought, on Which are Founded the Mathematical Theories of Logic and Probabilities." in Classics of Mathematics, Ronald Calinger, ed. New Jersey: Prentice Hall, 1995. Pages 95-96.

- The students are organized into pairs for a warming social component and to help each other think through making circuits. The teacher explains the first exercise is to build a circuit to be used to manually turn an LED on and off, and demonstrates the circuit. There is no intention here to volunteer how the components work, just to demonstrate the behavior phenomenologically. When Saratoga students asked questions I described the materials and a summary of the behavior in context of other components. The teacher hands out a physical diagram for the circuit with a component list (e.g., 1 breadboard, 1 4.5V power supply, 1 LED, 1 1500 ohm resistor, box of jumper wires). The student pairs pick up their components and make the circuit work as was explained and demonstrated. Some LEDs blow out because the resistor is not connected in series. The class gathers in attention to be sure everyone thoroughly understands what they have done. A schematic is handed out and the interpretation and usefulness of this abstraction discussed. A truth table is derived through class discussion.
- The teacher explains the second exercise is to build a circuit to use a transistor as a switch, and demonstrates the circuit. There is no intention here to volunteer how the transistor works, just to demonstrate the behavior phenomenologically. The teacher hands out a physical diagram for the circuit with a component list (1 more 1500 ohm resistor and a transistor). The student pairs pick up their components and make the circuit work as was explained and demonstrated. The class gathers in attention to be sure everyone thoroughly understands what they have done. A schematic is handed out and the interpretation and usefulness of this abstraction discussed. A truth table is derived through class discussion.
- The teacher reminds everyone of the AND function, hands out a component list, physical diagram, schematic, and truth table for the circuit. Each pair builds the circuit. Those who finish first can help those struggling.
- Similarly, OR and NOT are implemented. The circuits are labeled with the owner’s names. 3 ANDs, 7 ORs, and 6 NOTs, are required for a 2-bit adder.

Overview of content for Day Ten.

- ORs and NOTs are combined pairwise into 6 NORs. The change of symbol for the schematic is noted. The ownership of circuits is combined. Student pairs are now relying on the correctness of the circuits by other student pairs for the combination to work.
- The teacher reminds everyone the intention is to do arithmetic with these circuits even though how to do so accurately is mysterious so far. The teacher gives an introduction to the binary representation of numbers, works through exercises in addition of 1-bit numbers, and translates the truth tables so far into 1 and 0 format. A Boolean function operates on one or more inputs, producing an output. Each of the inputs and the outputs can have one of 2-states. These 2 states can be expressed as true and false or equally well as 1 and 0. By choosing 1 and 0 we can do arithmetic functions.
- The students are asked as a class for suggestions to help construct a truth table for a circuit that, if we could build it, would perform addition of two 1-bit numbers, e.g.

Input switch A closed	**Input switch B closed**	**Red LED on**	**Green LED on**
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0

  The class is now working out of this shared intention.
- The teacher demonstrates how 2 NOR circuits and 1 AND circuit are combined to implement the needed truth table. This circuit is called an "adder" – it is conventionally called a "half adder" but that name might seem confusing at this stage.
- The block diagram is introduced and the higher level of abstraction than the earlier schematics acknowledged.

Overview of content for Day Eleven.

- The students combine NOR and AND circuits into 3 adders.
- The students are asked as a class for suggestions to construct a truth table for a circuit that, if we could build it, would perform addition of two 2-bit numbers, e.g.

Input switch A closed	**Input switch A closed**	**Input switch B closed**	**Input switch B closed**	**Yellow LED on**	**Red LED on**	**Green LED on**
1	1	1	1	1	1	0
1	1	1	0	1	0	1
:	:	:	:	:	:	:
0	0	0	0	0	0	0

  The class is now working out of this shared intention.
- The teacher demonstrates how 2 adder circuits and 1 OR circuit are combined to implement into a larger circuit (conventionally called a "full adder") that when combined with an adder gives the desired behavior. Saratoga students built exactly 3 ANDs, 7 ORs, and 6 NOTs so we concluded with the delightful situation that for the 2-bit adder to work then each individual circuit must work and they must be correctly combined. Everyone was relying on everyone else's contribution and working together to make the whole functional.
- If the students are ready, the extensibility to a 3-bit adder and so on using multiple full adders is outlined.
- Analogies are made to memory, processing, and data input/output in a modern electronic computer - this is preparing the way for an explicit treatment of stored program control in 11th grade.
- A concluding perspective on transistors is given. Transistors have been made smaller, increasingly dense in their interconnection, and as a consequence of both developments also faster, cheaper to make, and cheaper to run. Interconnection of transistors was historically done at first using wires just like the work done with the students, then on a circuit board whereon are mounted discrete transistors plus wiring deposited on the board, then by an integrated circuit wherein multiple transistors and their linkages are manufactured onto a single chip of silicon. A well known chip, such as the Pentium, is described in terms of millions of transistors. Moore's Law is characterized. All these stages of evolution are exemplified with physical examples.

Overview of content for Day Twelve.

- Electronic calculators are identified as being composed of circuits like the adders just constructed.

- Detail is given here for an exercise with calculators. A practical issue arose in the Saratoga class in that many students could not properly input exponents; the syntax for that operation had to be understood for each type of calculator before the intended exercise could be launched.
- Half the class is asked to calculate [(3 + 9x10**-20) - 3] x 10**21 – 3, yielding –3.
- The other half is asked to calculate [(3 – 3) + 9 x 10**-20] x 10**21 – 3, yielding 87.
- The students are asked to compare the two expressions, and they are found to be equivalent; further, as can be computed even mentally from the second expression, the correct result is 87.
- The two halves of the class are asked to swap calculations. The same, wildly different, results occur. Saratoga students were perturbed and asked "why?" Faulty calculators cannot satisfactorily explain this phenomenon if there is a diversity of products. Faulty input by the operator would be unlikely to yield such consistent bimodal results. Saratoga students became more urgent with their question – the trusty calculator was behaving in a crazy manner.
- The intent for working with the soul capacities of a student for this day is as follows. By active doing with their hands, with both the calculator and paper and pencil, the limb activity stirs the will. Through the need to figure out the crazy results the student's will stirs her thinking. Thinking in turn enters her will to give intentionality to her nimble fingers, e.g., are these results for real? Following the hands-in work, an imaginative recollection of the facts stirs her feelings in relation to the hands-in work. These feelings are the fundament needed for the next step.

Overview of content for Day Thirteen.

- The intent for this day is that, following sleep, pictures are present and available to the thinking. Reflections and discussions are possible that, through inspiration, bring to consciousness in the child the rules embodied by the calculator that stand behind the observed facts from yesterday. This is the context within which the teacher presents the concepts of number representation capacity, precision, and accuracy. Will, which can draw on the precedent of the hands-in work, enters her thinking to form, direct, and connect these thoughts. By deferring until this day the conjoining of concept with the percepts from the previous day, a practice for "living thinking" is cultivated.
- The teacher guides the students to understand the representation of numbers by their machine, e.g., 1+10**-1 = 1.1, 1+10**-2 = 1.01, …., 1+10**-9 = 1.000000001, 1+10**-10 = 1. What happened to the 10**-10? Each calculator has a cut off at roughly this order of magnitude. Saratoga students guessed that limited memory <u>capacity</u> led to this loss of <u>precision</u> of representation of the number.
- Is there a rounding or a truncation? We explore the 10**-10 boundary. We cannot make the desired distinction when the last digit is less than 5, so 1 + 6x10**-9 = 1.000000006 is computed. Next, 1 + 6x10**-10 = 1.000000001 shows a rounding up. Further investigation shows that while the external display shows 1 + 6x10**-10 = 1.000000001 the internal view is 1 + 6x10**-10 = 1.000000006. This subtlety around internal representation versus external representation could confuse and take the focus off the overall process of truncation. Next, 1 + 6x10**-11 = 1, showing a truncation. This truncation is confirmed to have happened internally as well as externally. The 6x10**-11 has been ignored.
- A step-by-step display of the interim results for each of the two original calculations shows how the loss of precision of representation of the number led to a loss of accuracy.
- How to avoid such loss of accuracy? Well, we can increase precision by increasing the capacity for number representation. While that is helpful to solve known cases, the phenomenon can reappear as numbers arise that incur truncation. We must act decisively through our thinking, specifically through our understanding of algorithms, to "condition" the calculation to be "well-behaved". The students look at a specific practical example in detail. For example, consider an electric power company that calculates the network power loss as the difference between the power sent out into the network minus the sum of the power consumptions of buildings. The latter can be measured by a truck systematically driving the streets using a wireless reader of household meters - a talking point for Saratoga because this technology has just been introduced. After the first 15,000 readings, say, the total might be so high that each individual addition would be truncated, thereby not including the other 15,000 readings. The apparent loss would be huge, with many business and environmental implications. The summation has to be conditioned, e.g., measure each neighborhood and store the number, then as a separate step add the neighborhood totals which are likely to be sufficiently close to avoid truncation.
- Saratoga students felt that the calculator results made sense again, but the calculator could no longer be blindly trusted as before. The calculator had been shown to not embody the commutative law of arithmetic. Aware of the limited precision and its potential affects, the operator must retain responsibility and accountability for those, e.g., the –3 result was precise but inaccurate. Feeling in thinking is required to judge accuracy. This is supported in quantitative work by estimation, e.g., is the kind of number to be expected a small negative number or around 100?

- Transparency of operation (how easily can an individual understand the operation), power (speed and scope) of operation, limitations of operation (capacity, precision, AND accuracy), and extent of human delegation (e.g., memory, processing, ….) are summarized.
- The course culminates with each student summarizing their observation of trends in how people relate to computing machines throughout history:

	Transparency of operation	**Power (speed and scope) of operation**	**Limitations**	**Extent of human delegation**
Abacus				
Napier's rods				
Human computing team				
Electronic adder		X	X	X
Electronic calculator				

X means the cell was not filled out due to insufficient experience.

- For Saratoga students, the nature of the responses were generally along the following directions:

	Transparency of operation	**Power (speed and scope) of operation**	**Limitations**	**Extent of human delegation**
Abacus	Highest	Very low	Many	Least
Napier's rods	High for x and /, low for $\sqrt{\ }$ and $\sqrt[3]{\ }$	Low	Some	High
Human computing team	None for a 3rd echelon person	High	Few	Very high for a 3rd echelon person
Electronic adder	Low	X	X	X
Electronic calculator	Very low	Very high	Few	Very high

X means the cell was not filled out due to insufficient experience.