For Teachers

Conferences and Seminars on

Arithmetic

with Zoological Considerations

with

Dr. Karl König

Unedited: For Study Purposes Only

Arithmetic

and Zoological Considerations

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THE ARITHMETIC CURRICULUM

Lecture by Hans Christoph Valentien

Dear friends, when we teach arithmetic to mentally handicapped or disturbed children, our familiarity with the subject covers more or less the beginnings of number-work only. It is worthwhile to turn to the Waldorf curriculum in both arithmetic and geometry, especially in the light of Dr. König's lecture this afternoon, where he described mathematics as being rooted in the formative forces working in man's sensory organization, particularly in his lower senses. These forces become conscious in arithmetic and geometry.

Very important, just to us who are working in curative education, are certain general aspects of the Waldorf curriculum and of mathematics in particular; for instance, the question of how to introduce arithmetic and geometry to children in such a way that we reckon with human nature and life in their totality.

Let us ask, first of all, how arithmetic is placed within the whole of Waldorf education. As curative teachers we are aware of the curative, therapeutic aspects of the curriculum. Often we have experienced its harmonizing and healing value. In trying to adapt the Waldorf curriculum to the more limited scope of schooling in curative education, our first objective must be to retain and make use of, and not to lose, its therapeutic powers.

This objective is very much in keeping with the overall idea of Waldorf education. Remember that in the introductory courses given to the first College of Waldorf teachers, "Allgemeine Menschenkunde" (Study of Man) and "Methodisch-Didaktisches" (Practical Course for Teachers), Rudolf Steiner points outs: "The aims of Waldorf education are derived from the spiritual tasks of our age. In order that men can fulfill those tasks, the method of teaching in the widest sense should always have the aim of harmonizing the 'upper' and the 'lower' man, the spirit-soul and the body (Leibeskörper). In Waldorf education, any subject, be it botany, history, gardening or shorthand, is to become a tool in the hands of the teacher with this aim of harmonizing body, spirit, and soul.

Arithmetic and geometry, too, can work powerfully towards this harmony. Educationally they are in the first place not knowledge to be learned, but tools. How does arithmetic work upon the child? One of the first things we are meant to learn as Waldorf teachers is the following: In every subject we bring to our classes we must distinguish between 'conventional' knowledge on the one hand, and on the other hand knowledge which is based on the understanding of man's being, spiritual and physical. Conventional knowledge as Rudolf Steiner broadly defines it, is founded upon human convention, for example, reading and writing. Our alphabet is a convention. Conventional knowledge works only upon the head-nature of man, whereas knowledge of man, of his spiritual nature, links us to the spiritual world. Further, we read in ‘Methodisch-Didaktisches’ (all of this is in the first lecture):

> *Wir unterrichten im Gebiete des Allerphysischesten, indem wir Lesen und Schreiben unterrichten.*
>
> Teaching reading and writing we teach in a realm that is most physical. That is teaching conventional knowledge.

> *Wie unterrichten schon weniger physisch, wenn wir Rechnen unterrichten.*
>
> We teach already less physically when we teach arithmetic.

> *Und wir unterrichten eigentlich den Seelen-Geist oder die Geist-Seele, indem wir Musikalisches, Zeichnerisches und dergleichen dem Kinde beibringen.*
>
> And we actually educate the soul-spirit or the spirit-soul when we bring near to the child that which is of an artistic nature—music, drawing, etc.

Now follow very significant sentences:

> *Nun können wir aber im rationell betriebenen Unterricht diese drei Impulse des Überphysischen im Künstlerischen, des Halb-Überphysischen im Rechnen, und des Ganz-Physischen im Lesen und Schreiben miteinander verbinden, und gerade dadurch werden wir die Harmonisierung des Menschen herverrufen.*
>
> In all education that is handled rationally these three impulses—the spiritual in the artistic, the half spiritual-half physical in arithmetic, the physical in reading and writing—can be combined with each other, and it is just thereby that the harmonizing of man is called forth.

Can you already see the mediating role arithmetic can play if it is handled rightly? Everything artistic works upon the will-nature and involves the whole being of man. The conventional works on his head-nature only. Therefore:

> *Wir müssen auf künstlerische Art Konventionelles lehren in Lesen und Schreiben, wir müssen den ganzen Unterricht durchdringen mit einem küinstlerischen Element.*
>
> We have to teach creatively, artistically, the conventional in reading and writing; we have to permeate all education with an artistic element.

Arithmetic, however, stands midway between art and convention, as the soul stands midway between spirit and body. Speaking of subjects that work upon the physical and etheric bodies (such as drawing which leads to writing, and the study of plants) and of other subjects that work upon ‘that which leaves the physical body and the etheric body during sleep’ (e.g. the study of animal and man), Rudolf Steiner goes on to say:

> *Rechnen Geometrie spricht zu beiden. Das ist das Merkwürdige. Und daher ist wirklich in bezug auf den Unterricht und die Erziehung Rechnen sowohl wie Geometrie, man möchte sagen, wie ein Chamäleon; sie passen sich durch ihre eigene Wesenheit dem Gesamtmenschen an. Und während man bei Pflanzenkunde, Tierkunde, Rücksicht darauf nehmen muss, dass sie in einer gewissen Ausgestaltung in ein ganz bestimmtes Lebensalter hineinfallen, hat man bei Rechnen und Geometrie darauf zu sehen, dass sie durch das ganze kindliche Lebensalter hindurch getrieben werden, aber entsprechend geändert werden, je nachdem das Lebensalter seine charakteristischen Eigenschaften verändert.*
>
> Arithmetic and geometry speak to both. This is so remarkable. And therefore, arithmetic and geometry are, as regards education, like a chameleon; through their own essence they adapt themselves to the entire human being. And, whereas with botany and zoology we have to consider that they are to be given in a quite definite form at a definite age, with arithmetic and geometry we have to see to it that they are practiced throughout childhood, but changing in form according to the changing charac-teristics of the age, of life.

After having considered the place of arithmetic within education, and having seen that it actually arises in human nature, we realize that the Four Rules are not foreign to our human faculties in the way the letters of the alphabet are foreign to it. We are, however, also aware that an intellectual element can very easily be given to the child much too early; and a great deal may be spoiled in the child’s development. The child should be introduced to mathematics in a way “which can only be decided upon by one who is able to survey the whole of life from a spiritual point of view.”

The introduction of the Four Rules, namely addition, subtraction, multiplication and division, is prescribed for the First Class but in a rather unusual way, as we know. In contrast to everyday usage, the process of addition is to be brought near to the child’s understanding by proceeding from the sum, or total, to the parts—just the opposite of what is usually being done. Similarly, multiplication is to be introduced by proceeding from the product. But still more obscure may we find the indications regarding subtraction (to start with the remainder) and division (starting with the quotient). Only after this special introduction of the four processes, only after having secured a glimpse of analytical under-standing, should the Four Rules be practiced in the form in which they are commonly used: synthetical, beginning with the parts, and finding the total or product, etc.

Now arises the question: What is the meaning and advantage of the analytical way of introducing the Four Rules? The advantage is not necessarily greater skill in arithmetic, but is of a moral nature. That is surprising. Rudolf Steiner often spoke about the relationship between arithmetic and morality. In the course for teachers given at Oxford (1922) he says:

> In this way (the analytical way) we get the child to enter into life with the ability to grasp a whole, not always to proceed from the less to the greater. And this has an extraordinarily strong influence upon the child's whole soul and mind. When a child has acquired the habit of adding things together, we get a disposition which tends to be desirous and craving. In proceeding from the whole to the parts, and in treating multiplication similarly, the child had less tendency to acquisitiveness, rather it tends to develop that which in the Platonic sense, the noblest sense of the word, can be called considerateness, moderation. And one's moral likes and dislikes are intimately bound up with the manner in which one has learned to deal with numbers.

A little further on Rudolf Steiner concludes:

> Thus what comes to pass in the child's soul by working with numbers will very greatly affect the way ho will meet us when we want to give him moral examples, deeds and actions for his liking or disliking, sympathy with the good, antipathy with the evil. We shall have before us a child susceptible to goodness when we have dealt with the teaching of numbers in the way described.

One more statement I would like to quote in this connection, which can make us aware of the immense importance Rudolf Steiner attached to the proper use of mathematics in school:

> If, then, man had known how to permeate the soul with mathematics in the right way during these past years, we should not now have Bolshevism in Eastern Europe.

The first part of this remarkable statement about communism, namely that men did not know how to permeate the soul with mathematics in the right way, brings us to another equally important aspect. It concerns the teacher himself, his obligation to know what he is doing and why he is doing it. In the background of teaching arithmetic, particularly when doing number-work with the little ones in the First Class, the teacher should have an awareness of the process that leads man to understanding, to cognition.

This afternoon. Dr. König spoke of the realm of mathematics, of number, which is always latent in us, but in its entirety. Our experience of this living mathematics is not conscious. How does it become conscious? It certainly does not enter our consciousness as a totality, but in bits and pieces. How is it that the unconscious experience of the realm of number, the experience of the lower senses, is then split up, differentiated into so many facts of knowledge? Why this breaking down of a totality into single numbers, concepts and rules? While still a very young man, Rudolf Steiner wrestled with this question, but in a much more general form, not concerning mathematics alone, but all human cognition, and he answered it with flawless clarity in his early philosophical writings. Thinking, namely, has a twofold task. First an analytical one: As intellect it has to create single concepts with sharp outlines. Secondly, a synthetical one: as reason, thinking combines these separate concepts into a uniform, harmonious whole. Thinking's intellectual, analytical function is one that draws dividing lines, discriminates, holds on to differences; and only those 'bits and pieces' really become conscious to us which we have singled out, separated off, from our pre-conscious experience of the whole world. Already when we are given up to observing some external phenomenon, particularly visually, may we realize this analytical, discriminating, singling-out faculty. The reasoning function of thinking is one that unites and overcomes the separateness which is the only thing our analytical faculty can see in the world. The analytical side of thinking is called in German Verstand, the synthetical one Vernunft.

This philosophical aspect of the introduction to number work is gathered up in the following passage from the chapter "Verstand und Vernunft" in Rudolf Steiner's Grundlinien einer Erkenntnis—theorie der Goetheschen Weltanschauung ("Intellect and Reason" in *Fundamentals of a Theory of Knowledge*).

> *Die Einheit welche die Vernunft zu ihrem Gegenstande macht, ist vor allem Denken, vor allem Vernunftgebrauch gewiss; nur ist sie verbergen, ist nur der Möglichkeit nach vorhanden, nicht als faktische Erscheinung. Dann führt der Menschengeist die Trennung herbei, um im vernunftgemässen Vereinigen der getrennten Glieder die Wirklichkeit vollständig zu durchschauen.*
>
> The unity which reason takes as its object is there before all thinking, before all use of reason; only, it is hidden; it is there as a potentiality only, not as an actual phenomenon. Then the human spirit brings about separation in order to have a complete grasp of reality through the reason's unification of the separated parts.

The intellect, the power of analysis, brings us from the hidden unity to a world of separate facts. Reason, the power of synthesis, leads us from seemingly unrelated facts to the unity of the world-consciously.

> *Die Vernunft bringt die höhere Einheit der Verstandesbegriffe zum Vorschein; die der Verstand in seinen Gebilden zwar hat, aber nicht zu schen vermag.*
>
> Reason brings the higher unity of the intellectual concepts into evidence; the intellect has this unity in its concepts but cannot perceive it.

The procedure of introducing the child to number is obviously modeled upon this fundamental interplay of our soul forces. And it is on us to bear this in mind when we teach arithmetic, particularly in the lower classes.

In order that we better understand Rudolf Steiner's indications for the first introduction to the Four Rules, I would now like to show you that those four belong to a whole organism of 'rules.' On three levels we have three operations each, all together 3 x 3 = 9. Addition and multiplication, of course, do not lie on the same level. The question is, how do we get from the one level to the other in an organic way, how from the first to the second level—and further from the level of multiplication still higher to the rule of powers. Subtraction lies on the first level together with addition, and division on the second, with multiplication. On the third level we find powers, roots and logarithms.

We can start with definite numbers, say 3 and 4, then we do not become too general, too abstract. The first stage would be adding: 3 + 4 = 7. It is an enhancement of addition when we add together not only two, but three or four numbers—as many as you like:

$$3 + 4 + 2 + 8 + 1 = 18.$$

From this enhanced process of adding several different numbers, we take another step if all the parts are the same number:

$$3 + 3 + 3 + 3 + 3 = 15.$$

This leads us almost to multiplication.

The new idea which brings us to the second stage, that of multiplication, is to count the equal parts: How many 3s are there to be added up? We then say 5 x 3 instead of 3 + 3 + 3 + 3 + 3. The 5 is the multiplier, the 3 is called the multiplicand.

And again—via enhancement and transition—we advance to the third stage. We can multiply together more than two numbers: 5 x 3 x 8 x 2 = 240. This is an enhancement, a widening of the rule of multiplication. Now all the factors could be equal: 5 x 5 x 5 x 5 = 625. Here we are prompted to find out how many equal factors there are—and we come to a new concept, namely to that of power: 54 = 625 (5 raised to the power of 4, or the 4th power of 5). The index 4 tells how often 5 is a factor.

Stage I	$3 + 4 = 7$	Addition	$p + a = r$
	$3 + 4 + 2 + 8 + 1 = 18$	Enhancement	
	$3 + 3 + 3 + 3 + 3 = 15$	Transition to …	
Stage II	$5 \times 3 = 15$	Multiplication	$a \times p = r$
	$5 \times 3 \times 8 \times 2 = 240$	Enhancement	
	$5 \times 5 \times 5 \times 5 = 625$	Transition to …	
Stage III	$5^4 = 625$	Raising to power	$p^a = r$
	p: passive	a: active	r: result

These then are the three main operations and any two numbers combine on all three levels. Take 2 and 3. According to the rule of addition we have $2 + 3 = 5$; according to the rule of multiplication: $3 \times 2 = 6$; and according to the third rule, the index rule: $2^3 = 8$.

You know that for the sum-total of two numbers or for their product it does not matter in which order the numbers are added or multiplied together: $4 + 3 = 3 + 4$ and $3 \times 4 = 4 \times 3$. For the index rule it does matter, however, for the third power of 2 is not equal to the second power of 3: 23 = 8, but $3^2 = 9$. Even if two processes have the same result, as 3×4 and 4×3, this does not mean that they are the same. The sign of equality here ($3 \times 4 = 4 \times 3$) does not stand for the processes but only for their end results, namely 12. The processes are different: 3×4 means $4 + 4 + 4$, and 4×3 means $3 + 3 + 3 + 3$.

Evidently the two factors of a product are not of the same quality. One plays a more passive role, the other a more active one. The same can be said of two numbers forming a power. In 23 the 2 is the passive, the index 3 the active element that gets things going. Although it is not so obvious, but even in the process of addition we find that the two numbers play different roles. Take again $3 + 4 = 7$. The 3 has a static, passive role, and the + 4 is active; it is what actually proceeds from 3 to the total 7.

Let us use (p) for passive, (a) for active and (r) for result.

I.	Addition	$3 + 4 = 7$	$p + a = r$
II.	Multiplication	$3 \times 4 = 12$	$a \times p = r$
III.	Raising to power	$3^4 = 81$	$p^a = r$

On each level, the passive element (p) represents the number of departure, and the active element (a) bridges over to the number of arrival, the result (r). The result is again of passive character as is the beginning number of the operation. This becomes very clear when we multiply a compound quantity: 3×4 apples = 12 apples. Multiplicand (4 apples) and result (12 apples) are just passive apples. The decisive multiplier (3) is always a pure number.

All processes we so far considered have one thing in common: two numbers combine to produce a definite result. But they are differently related to the result:

a

p ———————————————— r

Number (p) stays put, as it were, marking the one end of the operation. Number (a) determines the span of the operation leading to the other end (r). The nature of the span is either according to the rule of addition or of multiplication, or according to that of power. Thus each process presents to us an interrelationship of three numbers, p, a, and r. If two numbers of such a relationship are known, the third can be found. There are three possibilities:

1. We know p and a; we want to find r.

Examples: $5 + 3 = ?$
$3 \times 5 = ?$
$5^3 = ?$ (p = 5, a = 3)

a

p ———————————————— r = ?

Given: p and a; wanted: r = ?

2. We can also assume that p and r are known, and ask: Which number a leads from p to r? We know p and r; we want to find a.

Examples: $5 + ? = 8$ (What is to be added to 5 to get 8?)
$? \times 5 = 15$ (By which number must 5 be multiplied to arrive at 15?)
? x 4 apples = 12 apples (How many times 4 apples makes 12 apples?)
5? = 125 (Which power of 5 makes 125?)

a = ?

p ———————————————— r

Given: p and r; wanted a = ?

Here we meet a first kind of reversal operation, reversal because we already know the result (r) and have to trace the way backwards to the other end (p) in order to find the active number (a). This kind of reversal is called active because the active element is to be found.

3. With the passive reversals, on the other hand, we are to find the passive element (hence passive reversal). Again, two numbers of the process are known: the result (r) as well as the active element (a). Where does the process begin? We know a and r: we want to find p.

Examples: $? + 3 = 8$
$3 \times ? = 15$
$?^3 = 125$

What all three passive reversals have in common can be shown schematically:

$$p = ? \overset{a}{\rule{10em}{0.4pt}} r$$

Given: a and r; wanted: p = ?

We have now all the nine operations together:

I. Addition

$p + a = ?$

$5 + 3 = ?$

Finding the Difference	Subtraction
$p + ? = r$	$? + a = r$
$5 + ? = 8$	$? + 3 = 8$
$r - p = ?$	$r - a = ?$

II. Multiplication

$a \times p = ?$

$3 \times 5 = ?$

Finding the Ratio	Division
$? \times p = r$	$a \times ? = r$
$? \times 5 = 15$	$3 \times ? = 15$
$r : p = ?$	$r \div a = ?$

III. Raising to Power

$p^a = ?$

$5^3 = ?$

Finding the Logarithm	Finding the Root
$p^? = r$	$?^a = r$
$2^? = 125$	$?^3 = 125$
${}_3\log 125 = ?$	$\sqrt[a]{r} = ?$

In the left-hand column of the table are the active reversals of addition, multiplication and raising to power; on the right are their passive reversals. What are they usually called? The answer becomes obvious if we write the problems in the usual notation. First the passive reversals.

I. $? + a = r$ becomes $r - a = ?$
$? + 3 = 8$ $8 - 3 = ?$ This is subtraction, or finding out how much remains.

II. $3 \times ? = 15$ becomes $15 \div 3 = ?$
$a \times ? = r$ becomes $r \div a = ?$ This is division (finding the quotient).

III. The passive reversal of raising a number to power, is finding the root of the power.

$?^3 = 125$ becomes $\sqrt[3]{125} = ?$ (what is the 3rd root of 125?),
$?^a = r$ becomes $\sqrt[a]{r} = ?$ (what is the a-th root of r?).

The active reversals, on the other hand, represent the three ways of comparing two numbers p and r. Here we are concerned with finding their difference, their ratio or their logarithm.

I. $p + ? = r$ becomes $r - p = ?$ (difference)
II. $? \times p = r$ becomes $r : p = ?$ (ratio)
III. $p^? = r$ becomes ${}_p\log r = ?$ (logarithm)

Here, in the active reversals of addition, multiplication, a.s.e., we meet again: difference, ratio (Dr. König used the word ‘relation’) and logarithm. They correspond as Dr. König described today to the quality of the experiences of the lower senses:

difference – sense of life
ratio – sense of movement
logarithm – sense of equilibrium

We have gained more background and can turn again to Steiner’s indications regarding the introduction to arithmetic. Translated into the terms we have been using here, Steiner advises in short:

> To introduce the arithmetical processes to the child, start with the result and the passive element and let the child search for and discover the active number, the transitory and more abstract element that leads over from the one to the other.

Here is a brief application of this general rule:

Addition p + a = r. To introduce addition, start with p and r and let a be found: $p + ? = r$. How many are added to p so that the total is r?

Subtraction r – a = p. As with addition, start with p and r: $r - ? = p$. But here the question is: How many are taken away from r so that p remain?

Multiplication a x p = r. Start with p and r, let a be found: $? \times p = r$. How many times is p factor in r?

Division r ÷ a = p. As with multiplication, start with p and r, let a be found: $r \div = p$. How many times is p subtracted from r? Example: Here are 6 grapes. To how many of your friends can you give 2 grapes?

It is striking that the child should be asked to find always just the most elusive element. What is the significance? We have seen that the active element in, for example, the process of multiplication is the ratio between the two known numbers or quantities (p) and (r). In addition and subtraction it is the difference. In the case of raising a number to a certain power, it would be a logarithm (which we experience only in sleep, in our will—even though we may be wide-awake otherwise). The analytical approach to the Four Rules apparently is meant to link up with and appeal to the experience of the lower senses, to lift up the more or less unconscious experience of the body to greater awareness.

Only when arithmetic has been duly introduced in this way—with real quantities of beans and shells, of course—only when the child has come to a first inkling of what it is all about, is he to be given the duty to learn the tables by heart. And I do believe that normally the learning by heart must refer to the form used in everyday life:

$$2 + 3 = 5,\ 5 \times 2 = 10.$$

Generally there is no distinction made between the two reversals of addition, between remainder and difference; generally, the distinction between division (quotient) and measuring (ratio) is not clear. This is due to the fact that in addition and multiplication we may exchange the passive and active elements without affecting the result. But the process of reaching the result is, after the exchange, a different one. In stage III we may not exchange (a) and (p) without changing the result (generally speaking). Therefore there is no danger of mixing up logarithm and root. But due to the law of exchangeability (valid only for addition and multiplication) we speak of ‘the Four Rules’ instead of 6. Or, including the operations in stage III, of 7 rules instead of 9.

Counting also belongs to the curriculum of the First Class, and it is interesting that Rudolf Steiner speaks of the introduction to the Four Rules, assuming that the children have not yet learned to count. We would find, in fact, that the child is not so near to counting as he is to the Four Rules.

One reason for this is surely the conventional element in counting. Why do we count in 10s and not in 3s? Another reason is that the first tender grasp of number is not a strenuous matter for mind or memory, but is gained for instance in the absorbing activity of sharing out a heap of beans under the guidance of the teacher whose authority asserts: This heap I call ‘3 beans,’ that heap ‘7 beans.’ What is nearer to the child at this stage is to learn from the practical experience of the interplay of a few single number concepts—without stringing up and memorizing a whole sequence of as yet meaningless words.

A word still about measuring: It is an active reversal of multiplication. To measure means to compare two quantities of the same kind, to find their ratio. Nevertheless, measuring is similar to division and should also be introduced (in the Third Class) similarly. We measure the length of a classroom with a yardstick. However, for the introduction of this operation the problem should be put differently: To what length must we cut a stick so that the class-room is 15 'sticks' long? For in measuring, the unit quantity—one stick—plays the same role as the active number in division. The result, namely whether the classroom is 15 'sticks' long or only 10 'sticks' long, depends on the length of the stick. And a child who has learned to measure in this way will no doubt later find a truer and more independent appreciation of physics—knowing that all units of measuring such as yard, meter, pound, and so on, are conventions.

Arithmetic and geometry go like a red thread through childhood and have to change according to the development of the child. There are the two nodes in child development: the 9th–10th year and the 11th–12th year. I need not describe them. The curriculum shows corresponding changes, about which Rudolf Steiner speaks most clearly in the Christmas course of 1921. Before the 9th–10th year everything is brought near to the child in an entirely artistic and creative way, so that he partakes fully. Around the 9th–10th year we have, in addition to the creative element of art, a new objective in teaching; we now have to also describe everything more from without—for an inner distance between the child and the world around has sprung up and is increasing. Only towards the 12th year should education begin to complement the creative and descriptive elements by explanations which call more upon the awakening powers of thought, of intellect and of reason. Now we can speak with the children, for instance, about cause and effect.

Mathematics then confronts us with demanding and manifold problems in teaching. The two main tasks are obvious: first, that we bring mathematics in its various aspects throughout the school years; secondly, that we bring it so that it corresponds to the inner divisions in those years. The teaching of mathematics must correspond to the general characteristics of creative art, throughout this whole time: of description, starting from the 9th–10th year, and of explanation and proof beginning not before the 11th–12th year. For geometry this means to bring three almost self-contained courses.

Before the 9th year, geometry is held back, as in a bud, in a form that is contained entirely in the artistic drawing, in the walking of forms and so on. A strong sense of symmetry helps to develop the feeling for space. Between the 9th–10th year (Class Four) the transition is made to a more descriptive method. By drawing and describing, the child shall grasp the geometrical forms and their relationships. This course of 'visual geometry' is to lead as far as the Theorem of Pythagoras. Of special importance is the development of an awareness of space through movement games and shadow experiments. The third course, starting in the 11th–12th year (Class Six) takes up once more the content of the ‘visual geometry’ and introduces a more intellectual-reasonable mode of treating geometry.

For arithmetic a similar division must be found, even if one does not get much beyond the Four Rules. Considering the special tasks of curative education, a great deal remains to be worked out.

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Conference 1 – Seminar I

I would never have thought that a time was to come when I would have to speak about arithmetic. The only similarity which I have with Goethe is that we both don't understand anything about arithmetic and mathematics. Nevertheless Goethe has written a very interesting essay on his relation to mathematics, and I am not going to repeat it in now saying something about my relation to mathematics, but will try to relate certain parts of arithmetic and mathematics to human science. I know very well that it is my own fault that I have to stand here in front of you: because it was I who proposed to the Teachers' College that I should once speak to you about arithmetic and mathematics—and so it is now. What I had in mind then was exactly this: that I would like to find out something about this relation between man-who-is-a- mathematician and man himself. This is no doubt a great problem, and I know that you as teachers are continually struggling with this problem, that every day in the class you meet the difficulties of man the mathematician—and try to introduce the mind, the being, the existence of your children into this realm of mathematical thinking. We could also say: 'into the realm which is expressed by numbers.'

When visualizing this I thought we would have to come together and work out what man the mathematician looks like—what he is and what he is not—and that perhaps from this possible approach we might find certain ways whereby to awaken the minds of those children with whom we have to deal, in the realm of numbers. This is the first thing. And we can already look, for instance, at some types of our children: the mongols, who (when they are ordinary mongols) are hardly able to grasp what a number altogether means—to distinguish between 2 and 3 and 4 and 5 and 6 and so on—let alone to grasp, to understand, what 8 minus 2 means, what 2 times 4 actually wants to express. We find in epileptics, for instance, that their memory for numbers and number-work, their memory for mathematical operations, is exceedingly poor and that they have hardly the possibility to repeat from one day to the next what they have been told or taught in the previous week. You can find in autistic children that they have a very strange relationship to numbers, as if they would not think numbers but be able to handle them, to get about with numbers, to find results in mathematical problems and number-work. It is hardly believable; can we really understand what it is?

And as soon as we approach this, we already come to something very special, which I would have to say right from the beginning, whether you believe it or not. Each child, however retarded or backward he is, right down to those who are not able to move or to speak, as long as he is a human being, has a full idea of every-thing that is connected with number, mathematics and arithmetic. The world of mathematical ideas is present in every human being. This we should know. This is, so to speak, a mathematical axiom, from where we would have to start. All the children who sit in front of us are perfect representatives of man the mathematician, in so far as mathematics is present in. the world of ideas (and if I speak now of ideas, I mean the Platonic ideas). This world is open to every human being. But the task of the teacher—your task, every teacher's task—is to make this world of ideas, the 'world' (the sphere) of numbers, available to the child's mind. That means you are not going to start outside to tell him how much 2 and 2 is as if he didn't know. You can start outside—but you would have to know that you only do this in order to create an activity in the child's mind, whereby the present idea of number, the sphere of mathematics, comes to the child's vision. And. if I say 'vision,' I do not mean only vision. You as teachers have three possibilities for making mathematics available to the child, and these are the following: You can make it either visible or you can make it audible or you can make it touchable. Through vision, through hearing and through sense of touch, you have the means to bring the idea into the concept.

You can even bring the concept into the percept, as long as you are able to see: In mathematics we have to go an entirely different way, even an opposite way from other subjects which we have to teach. We come from outside, not with the essence but with the image, in order that the essence (the idea) appears, becomes concept, becomes percept, and is either visible or audible or touchable for the child. It is in three spheres—in the sphere of the visible space, in the sphere of the space of touch, in the sphere of the audible space—that the concepts and percepts of numbers appear. If we achieve this, we have made the necessary steps for teaching mathematics.

Now, this is first of all what I would like to put before you. I know we will have to make many steps together in order to under-stand all this in order to be able to come to grips with that which I tried now to put before you in a kind of outline. But let us go through this whole course with the idea that from outside we bring something to the child in order that the inner knowledge of mathematics that is present and available in him can more and more incarnate. And after we have made this statement, we should first of all ask ourselves: How does the mathematical ability, the arithmetic capability of the child, how does it come about? Some of you probably know the lecture-course of Rudolf Steiner: "*Grenzender Naturerkenntnis*" (*Boundaries of Natural Science*), which he gave from September 27 to October 3, 1920, during the opening of the Goetheanum. In the third lecture (on September 29) he actually does nothing else but try to make his listeners understand what mathematics is. And there he starts with the question:

Was ist es denn, was wir als eine Faehigkeit ausbilden, indem wir mathematisieren?

What is it actually, this ability to do mathematics?

This is the question. And then Rudolf Steiner points to the fact to which I have already pointed and have tried to draw your attention: that natural science—which we learn by experiment, by looking outside, by following up certain phenomena—this is something entirely different from mathematics. This is because (and so he tries to explain it) the mathematical faculty in us is present, it is here, though it is a latent faculty. It is not always available, but it is nevertheless present, in the same way, for instance (this is Rudolf Steiner's example), as warmth may not appear at certain times because it is latent warmth which has to be awakened. And when he then asks: 'When is it that the child transforms this latent ability to do mathematics into the active ability?' he gives a clear-cut answer. He says (and I quote now from this lecture):

> *Ist denn dasjenige, was Mathematisieren ist, im Menschen immer da, insofern er sein Dasein durchlebt zwischen der Geburt und dem Tode? Nein, es ist nicht immer da. Das Mathematisieren erwacht erst aus seinem latenden Zustand.*

He says quite clearly that the ability to do mathematics wakes up at a certain time. I go on so carefully because I have the impression it is of the greatest importance that we make these steps together. And then he says:

> *Wie allmaehlich diejenigen Seelenfaehigkeiten gewissermassen aus dem dunklen Untergrunde des menschlichen Bewusstseins heraus erwachen, die sich dann gerade in Mathematisieren und in Aehnlichem, was wie das Mathematisieren ist, von dem wir noch sprechen wollen, aeussern.*

There he says that the ability—and now something very important to me appears here—*die Seelenfaehigkeit*, the ability of the soul to do mathematics rises up out of the dark ground of the unconscious. The ability of the soul to do mathematics arises out of the dark ground, out of the darkness of the unconscious. Then he asks when, and says:

> *... dieser Zeitpunkt liegt etwa in derjenigen Lebensepoche, in der das Kind die Zaehne wechselt, in der aus den Milch zaehnen die zweiten Zaehne werden. Man muss nur solch einen Lebensentwickelungspunkt aus derselben Gesinnung heraus ins Auge fassen, wie man z. B, in der Physik gelernt hat den Schmelspunkt oder den Siedepunkt zu behandeln.*

So this is, as it were, a law that at the second dentition, when the teeth are changed, round about the seventh year (we would now have to say 'round about the sixth year'), there the abilities of the soul to do mathematics start to arise. And he says:

This is a point which is as clear and important as for instance the boiling-point and freezing point of some fluid substance. You see, such an example must be burned into our thinking, so that we are not going about just repeating, “When a child is seven years of age (anyway we know it), then his faculties for mathematics come about.” What we have to do is to wake up when such a statement is made, and begin, full of wonder, to ask: From where do these abilities for number-work arise, just at a time when teeth like an edifice are put into our mouths?

This in itself—and we will speak about it tomorrow—is already a mathematical problem: First we have 20 teeth—and at this moment when it happens we have 32 teeth (or gradually develop 32 teeth). This we should learn to understand. And Rudolf Steiner then says something which is still more important:

Ja, meine sehr verehrten Anwesenden, in dem Kinde bis zum 7. Jahre ungefaehr ist eine innere Mathematik, eine innere Mathematik, die nun nicht so abstrakt ist, wie unsere aeussere, sondern die kraftdurchsetzt ist, die, wenn ich diesen Ausdruck Platos gebrauchen darf, nicht nur angeschaut werden kann, sondern die lebensvoll taetig ist. Es existiert in uns etwas bis zu diesem Zeitpunkte, das mathematisiert, das uns innerlich durchmathematisiert.

This means that, before these abilities of number-work arise, there is an inner mathematics. But this inner mathematics is full of power. It is full of life. It is not as abstract as 1, 2, 3, 4, 5, 6, 7, but it is a living force in us, which builds and forms within our body. If we can only begin to realize this—not to know it (this is easy) but to realize it—that if we look at the little child, that within the growing child, within this child who learns to walk, to speak, to think, in this child who builds out his skeleton, muscles, who learns to do many movements, wakes up in speaking, that everywhere there the living powers of mathematics are inserted, work, form, mold, create, build, formulate arms and limbs, organs. And as soon as an edifice—the teeth—are built, some of these powers begin to become available, in order to make it possible for the ideas of number to appear as concepts and percepts.

And then Rudolf Steiner goes one step further. He asks the question: Where do these powers of mathematics come from? And he has a clear-cut answer. He says that they arise out of the three lower senses. They arise from the sense of life, from the sense of movement, and from the sense of equilibrium; so that, so to speak, these living forces of mathematics—these living abilities of counting and reckoning—they work in the sensory organs of life, of movement, of equilibrium. And after these three senses are partially built up, these forces become available within the human mind. But these are not—and may I make this clear to you—these are not etheric powers. They are powers of the soul. We might also say these are the powers of the astral body. You see Rudolf Steiner describes very extensively, in the lectures to teachers, how etheric powers form our organs, form our tissues, form parts of our body, and as soon as they arise, after the second dentition, they become powers of thought.

On a different level something similar takes place with the powers of mathematical thinking, with the powers of arithmetic. But these are soul powers. They are powers which are not connected with the formation of organs and tissues, they are powers which are intimately inserted into sensory organs and not living organs, into nervous substance: sensory-organ substance, and not ordinary tissues and ordinary organs. If you make this division, you already begin to under-stand that we have one sphere, and this is the sphere of our thinking. This sphere is not identical with the sphere of mathematics. The sphere of thinking is permeated by the sphere of numbers, is permeated by the sphere of mathematics. And through number-work and mathematics, within our thinking we become aware of its pattern. We will still speak about this.

Now we have gained a certain step. We know now that we cannot throw together thinking and mathematics. We know we cannot throw together the living forces which build our organs during the first seven-years period, with these abilities and powers to do mathematics. The one are living powers, the other are soul-forces. And these soul forces work in the sense of life, in the sense of movement, and in the sense of equilibrium.

And we would now have to go one step further—it's the second step—and try to create a kind of real picture of these three spheres: of life, of movement, of equilibrium—not of the living forces but the sense of life, not of the movements but the sense of movement, not of the possibility to keep equilibrium but of the sense of equilibrium. I say this because time and again these two spheres are thrown together. To see is something entirely different from the image which we see, and to live is something entirely different from the sense of life (that means from the experience of that which lives in us). And what is the sense of life, or what do we know or experience? What does the sense of life give to us? The sense of life is concerned with the fundamental experience of our well-being. If I say this, you might say: 'Well, probably I have no sense of life, because I hardly ever feel well!' Never mind—we all have our sense of life. Only sometimes—rather seldom—are we permeated by this feeling of well-being. I could observe yesterday morning, for instance, how suddenly so many people, feeling spring in the air, had a very pronounced experience of their sense of life. They walked a bit more quickly, they spoke kindly, they opened their mouths when they saw each other: It was quite amazing to see such an amount of friendliness among you—it was the experience welling up through this scent of spring which the sense of life gave to you. At other times, and much more often, this well-being is disturbed by many different kinds of pains and aches and discomfort, by feeling unwell, by tiredness, hunger, thirst and God-knows-what. The sense of life is a kind of oneness in feeling well. This can rise; it can also descend. But this feeling of oneness is continually disturbed by manifold different experiences, which disturb this beautiful crystal-clear surface which gives us at least the feeling of being. This is disturbed by pain, by a mood, by this or that. It is quite necessary, because it keeps us awake to the experience of the life which lives, exists, in our body. And if you ask what do we experience, in point of fact, with the sense of life, I would have to say it is nothing else but the continuous equilibrium between the powers which build up and the powers which destroy our organization. This is a concept, a coming and going, not hundred-fold but million-fold. The powers which are building up, growing and the powers which are destroying, which are destructive, which are the powers of death. And you know that Rudolf Steiner once put before us a kind of picture. He said: Imagine a huge heap of earth, and every day we put on a certain amount, but every day another amount is taken away. And every day one shovel more is taken away than put on. So that the destructive forces from birth to death are always stronger than the creative forces. And so we will count up how long it takes until nothing of this heap is left—and then at last we are on the way to die. This experience of the shovels putting on and the shovels taking away, in a oneness, this is the sense of life.

The sense of movement is something quite different. I won't describe it in detail, because you know it. You know that through the sense of movement we have a continuous awareness of how the different parts of our body are towards each other: whether my fingers are stretched out or bent, my arm is bent or stretched—all this is sense of movement. And don't think that we could ever live, walk, work, sit, go about, lie down—without the constancy of the experience of the sense of movement. We know exactly, though this experience is a very low one; it is a dumb experience, it builds the foundation, the basic structure of our consciousness. I know how my fingers are placed, how my foot, my limb, my step, my elbows, my trunk, my head, my eyes, my ears, everything is placed. Can you imagine the manifoldness, every second, of this sense of movement! And it always flows together and is renewed, and flows together and is renewed, and continues to give us the constant experience of 'I am here. I am, within my body.' It's not only well-being which we feel, not at all. It is our insertion into our bodily frame which we feel. And if you ask 'Where do we experience this?' it is always there where the stream of our movement is either carried on or interrupted by our joints. Wherever a joint lies—and there are hundreds of joints all over—there the sense of movement comes to awareness. Not in the bone, hardly in the muscle: but where the muscle or the tendon touches a joint—there the sense of movement (and you will understand if I use this word) starts to flower and wither away, to flower and wither away. Thousands of times every hour, millions of times every day, billions of times through our life (I use these words consciously) the sense of movement lives in us, and we experience it. And the child, of course, during the first seven years, experiences this much more strongly. How happy a child is in walking, in moving, in speaking, in all kinds of mobility-activity—because there the sense of movement gives him the experience 'I am here.' But you see in all this, before the second dentition, mathematics (the powers of mathematics) they live there: They live in the sense of life, in the coming and going processes of life; they live in the million-fold flowering and withering, wherever our joints are. And if you still investigate how we move and how the sense of movement continually has to guide the movement, then you will find that every movement is born out of three different kinds of activity: of melody, of harmony, and of tact or rhythm. And this threefoldness of melody, harmony and tact (tact in the musical sense)— of this the sense of movement is aware. And there you already experience something, though probably not quite consciously, of the parallel—I would even say the identity of mathematics, music and movement. Sound, number, movement tact—this is one, this is soul, this is astrality.

MELODY

HARMONY

TACT

And now the sense of equilibrium. The sense of equilibrium is not only this sense which gives to us the possibility of gradually learning to stand upright and negotiate the powers of gravity. This is the skeleton of the sense of equilibrium. But the sense of equilibrium is not only a skeleton. It has muscles and organs, it has a heart, it has blood-circulation. It is a tremendously complicated power within our body of warmth. Some scientists, and not the worst ones, doubt altogether whether such a thing as the sense of equilibrium exists. In one way they are quite right because in ordinary life we hardly experience the sense of equilibrium. You see, the sense of life is rather in the foreground with regard to conscious experience. Lower, more numb, is the experience of the sense of movement. Almost completely dumb is the experience of the sense of equilibrium. We are hardly aware of it. Only in the moment when we are out of balance—when we stumble, for instance, or when we lose our grip, when we are ill and start to sway—then we suddenly become aware of how all possible powers in us are rallied together in order to establish the equilibrium again. As soon as we have gained it, the experience of it is gone.

But the sense of equilibrium is not only between above and below, between light and gravity. The sense of equilibrium lives in every breath, when we inhale and exhale. It lives in every heart-beat, between contraction and dilation. It lives in every process of digestion. That we experience 'in front' and 'back,' this is an experience of the digestive powers in us. They receive from in front and give away something from behind. This is front and back. And right and left as well is experienced, in lifting our arms, in our pulse-beat, in our breath, in our sympathy and antipathy. In all that—right and left, front and back, up and down—this is all experienced through our sense of equilibrium. And the sense of equilibrium does not only experience this: because it is an active sense, it also experiences and keeps order. It keeps, for instance, the equilibrium between acid and base in our metabolism, so that acid fluids are produced in our stomach, on the side of our liver, and of the duodenal, base fluids are produced. This is the sense of equilibrium. That in each part of the bone marrow billions of red and white blood corpuscles are continually created, and equal billions continually destroyed, and that a certain number of blood-corpuscles is kept in the flowing blood—all this is the sense of equilibrium. Building of blood, destroying of blood.

And those of you who have perhaps some notion of what differential and integral quotients are, you will suddenly discover that all this is nothing else but the living power which the sense of equilibrium continually creates in us. This is all here, it is within us. In the sense of life we simply count. We count the living forces and the destructive forces. We count the numbers of shovels which build our body and destroy our body, and it is not too much to say that here (in the sense of life) addition and subtraction are living. Continually we must add and subtract—especially must the little child, but also we ourselves—not here, but here, in a living way, in order to experience either a well or unwell feeling. Whatever we do (we will speak about this extensively tomorrow) through our sense of movement and experience in our sense of movement, this is a continuous multiplication or division. And so in the sense of equilibrium, there we come 'to raise to a higher power'—to potentize and 'to extract roots.' Raising to a higher power, extracting of roots—this you have with the sense of equilibrium, and then it goes, of course, still higher into differentiation and integration. There you already become aware of what I am after: to lead you into the living fountains of mathematics, the living fountains in us.

I would still like to go one step further for a few minutes, and to prepare for tomorrow. And this is the following. When we look livingly into the sense of life, I think we begin to realize—and again take the image of the earth—that in counting (in addition and subtraction) we are looking for nothing else but for the difference. Take for instance 8 and 2; the, difference between 8 and 2 is 6. Can you understand what I mean? This is the difference. In the sense of movement, it is not counting any more, it is not the difference any more. It is the ratio of numbers which matters, not the difference.

DIFFERENCE	$8 - 2 = 6$	sense of life
RATIO	$8 : 2 = 4$	sense of movement
LOGARITHM	$8 = 2 \times 2 \times 2 = 2^3$	sense of balance

And this relation of 8 and 2 is 4—but how different! So the first would be the sense of life, and the second would be the sense of movement. And if we go now to the sense of equilibrium, we already come to what we can generally call the logarithm. This is the sense of balance. And this means: 2 x 2 x 2—three steps—is $2^3 = 8$. Three entirely different forms of relationship. Three entirely, different ideas in the whole sphere of Number. And this must first of all be rooted in every teacher of mathematics: to know the difference between Difference, Ratio and Logarithm. You have to know that 2 + 6 = 8, that 2 x 4 = 8 that 2^3 = 2 x 2 x 2 = 8; and that each time—out of this number (8), in connection with the other number (2)—something entirely different is done. Within your thinking mathematical ideas begin to insert themselves.

And there is something very important to which I would like to draw your attention—something which I found when preparing for this Conference in a lecture of the late Professor Locher. He also speaks about these three: the Difference, the Ratio and the Logarithm, in connection with the three different types of comparison. And there he suddenly makes us aware that the difference is connected with our thinking; that the ratio (e.g. between a huge and a small pillar, a tall and a short man), that this relation is not a conscious experience any more, it is a kind of half-subconscious experience which lives more or less in the world of our feeling, our dream-consciousness. And that the logarithm—it always needs a tremendous amount of power to raise it up out of the unconscious—this is the sphere of our willing. And in this moment we again discover how thinking, feeling and willing—how sense of life (difference), how sense of movement (ratio), how sense of equilibrium (logarithm)—show the first traces of the connection between human science and mathematics.

This as a first step. I wanted to show you some of the great lines which I have in mind. Perhaps you will take it with you. And tomorrow we will continue, in going into more details. I still have something in mind, but I think it would be too much, because—apart from a few of you, who are studied mathematicians—it was perhaps a little bit stiff going.

Conference 1 – Seminar II

In our discussion yesterday, the question arose about the history of mathematical thinking. I indicated a lecture-course which Rudolf Steiner gave, "*Die tieferen Geheimnisse des Menschheitswerdens im Lichte des Matthaeus-Evangelium*" (*The Deeper Secrets of the Evolution of Mankind in the Light of the Gospel of Saint Matthew*), three lectures on November 1, 9 and 23, 1909, in Berlin. I do not know whether this is still in print or even translated, but tomorrow I will refer to it extensively.

What we tried to find out yesterday afternoon was, of course, a first all-round view about man who is able to do mathematics, man who is able to do number-work, man who is able to bring into his consciousness all that which we could describe with the word Mathematisieren— to do mathematics. And you will have seen that it is an extremely delicate thing to find out what it is when we count or when we add and subtract, multiply, do division. All these, of course, are such delicate things within the region of our mind, of our thinking, that it takes a great amount of intimate awareness to find out a little bit more about such processes, about such capabilities. As we tried to explain to each other (you in your way and I in my way), we found that man is a mathematician. One could almost say he is man because he has, besides many other possibilities and abilities, also the capability to do mathematics. He cannot only construct a cathedral, a house, a grave, a monument, but also count, construct it in mathematical signs and symbols and in numbers, so that out of this the work can be done. This is a different way from the way in which we develop it from childhood to youth, because we found that first mathematics is a living power, especially between birth and the second dentition. Only afterwards these living powers are liberated, freed, and by freeing and liberating these powers the child begins to do the third of the three Rs.

It was the most fundamental discovery, which we made together, that what is liberated is the power (part of the whole of the powers) of the astral body. And the astral body is that which ensures to us our movements. We would never be able to move if we only had a physical and an etheric body. We would be bound to the ground in the same way as the plants are. They are unable to move. They are moved by wind and rain and light and other powers of nature. But as soon as breath, astrality enters—in the self-same moment, movement begins. And in another lecture-cycle on natural history (in the one which Rudolf Steiner gave during the burning of the Goetheanum), he clearly says in one sentence: "*Und die ganze Mathematik, die ganze Geometrie, ist herauf geholt aus unserem Bewegungssystem* (And the whole mathematics, the whole of geometry, is raised out of our system of movements, out of our motor-system)." This is, of course, a most fundamental statement. And we would now have to ask ourselves (it was already asked yesterday afternoon, justifiably asked): How can we understand this? How will it be possible? And I think I regard this as one of my main tasks in these three seminar talks—to make us understand: mathematics and our astral body—movement. Out of the movement—Rudolf Steiner says it and I repeat it, because we must engrain ourselves with these concepts, so that they become living reality—out of the movement we raise the ability to do mathematics. And then we said: The astral body, it is movement. Is there a possibility to understand this a little bit better? I think there is a possibility if we again try to find certain indications which Rudolf Steiner has given, indications to which I have pointed time and again, but I think none of you has taken it so far earnestly enough. And therefore I am going to do it again. Only when we begin to have an understanding for this intimate identity (and I replace now the word 'connection' with the word 'identity'), if we gain an understanding of the identity of astral body and movement (motor-activity), and if we see that within the motor-activity—that is, within every movement—mathematics is secretly enshrined, enclosed, that every movement is as it were nothing else but an offenbares Geheimnis, a revealed secret of mathematics, and that this is astrality—only when we learn to understand this, not generally but in detail, will we begin to define the mathematical process: the working of mathematics in us, in nature, in the cosmos.

And Rudolf Steiner here says the following. I will always take several points together, because I don't regard this as a lecture but as a kind of seminar. And if you can't follow, please interrupt as often as you like. It should be a discussion. Rudolf Steiner says (on March 7, 1911) in the lecture cycle *Exkurse in das Markus-Evangelium* the following:

> *Es wird gar nicht viel Zeit dazu gehoeren, so werden die Menschen es einsehen, dass der Muskel allerdings nicht in Bewegung gebracht wird durch Nerven, sondern dass er in Bewegung kommt durch unsern astralischen Leib—und zwar durch das in unsern Astralleibe, was in diesem zunaechst nicht unmittelbar so wahrgenommen wird, wie es ist. Denn das ist ein Gesetz, dass das, was wirken soll, nicht unmittelbar wahrgenommen wird. Was den Muskel in Bewegung bringt, was irgendeine Bewegung des Muskels hervorruft, das haengt zusammen mit dem Astralleib—und zwar so, dass im Astralleib selber zur Bewegung des Muskels eine Art Tonentwickelung, eine Art Schall entwicklung stattfindet. Etwas wie eine Art Musikalisches durchdringt unsern Astral-leib, und der Ausdruck dieser Tonentwicklung ist die Muskelbewegung. Es ist wirklich so, wie wenn wir bei den bekannten Chladnischen Klangfiguren leicht beweglichen Staub auf eine Metallplatte bringen und diese dann mit einem Violinbogen Streichen: da bekommen wir eine Figur. Von lauter solchen Figuren—die aber Tonfiguren sind—*

ist auch unser Astralleib durchzogen, die zusammen bewirken, dass unser Astralleib eine bestimmte Lage annimt. Das ist eingepraegt in dem Astralleib.

Now I don't want to translate this sentence by sentence, but I would like to bring it to you so that you can understand it in your language, Rudolf Steiner says roughly the following: A time will come when we will have to begin to understand that the muscle is not moved by the nerve, but that all movement of the muscle is done—and there one should say the word directly—that all movement of the muscle is coming about directly through the astral body. Now I will interrupt and say something personal, and then we'll go on again. The astral body is directly inserted in the muscle. Our muscles need the nerves in order to let us know that they are moved. The so-called motor-nerves are partially (not altogether, but partially) the nerves for our sense of movement. That we can feel this which I tried to explain to you yesterday afternoon—how the different parts of our body are positioned towards each other—this is due to the motor-nerve. The motor-nerve helps us in our mind to receive the bending or stretching of the parts of our body. But the movement itself, however this movement be—gesture, speaking, walking, whatever—this is I, my astrality does it. And the muscles are nothing else but the instrument in order to achieve it. Now Rudolf Steiner goes on to say: If you observe the astral body in the act of movement, then you will find that each movement is done by a kind of sound, by a kind of tone, which appears within the astral body.

"*Und zwar so, dass im Astralleib selber sur Bewegung des muskels ... eine Art Ton entwicklung, eine Art Schall entwicklung stattfindet*."

Within the astral body, in the moment when we move, sound, tone, appears.

And we can now say—and please keep this—that the identity of movement and astral body is raised to the identity of sound and tone within the astral body and movement. And Rudolf Steiner still goes on to describe it in the following way, and this is also very helpful for all that which we try to understand. We should imagine now, just as outside the sound-figures of Chladni are coming about—in the same way we should imagine these figures, but figures of sound in our astral body. Different kinds of sound, different forms of sound, different characters of one or two or three modes, rhythms, tacts of melody appear. And what inside is music, outside is movement.

This astrality inside, music which we don't hear, sound which we can't experience, tones which arc unknown to us—they appear; not one or two or three but hundreds and thousands, they continually appear. If we would hear them we would be drowned, and we couldn't do anything else but listen to this sometimes harmonious, some-times completely unharmonious music. But it is there. And this music is identical with movement. So that we can say: Music, sound, is astrality, is movement. But out of this movement we raise that which we behold as mathematics. Is one to be astonished? Is it a wonder or is it not a matter of course, that if we study the laws of music we find nothing else but mathematics? Every rhythm, every melody, every tact—everything—the different heights and depths of sound and tones: It is not only to be expressed by mathematics. It is mathematics. And if you strip music of sound—which today, for instance in modern music, is more or less already done, and it remains nothing else but noise—you will suddenly see that a modern musician is a mathematician. He deals not in ideas only, but in different kinds of noises. But they are at least mathematical noises. I only say this in order to make you aware that we do not deal with different subjects. We deal with different expressions of the same thing. When describing music (I do it now in one term but you will find different terms) there are many possibilities of finding the fundamental laws of music, but the fundamental law is always (in the terms of art, in the terms of music, we could also do it in mathematics) that there are the three great underlying acts of music: Melos, which more or less (not in modern music, although it also contain melos) indicates melody; then Rhythm, and this is equivalent to harmony; and then there is Tact. [The nearest English equivalent to 'Tact' is 'beat.'] These are the three which we call in German *Bestimmungsstuecke* of music. Out of this every musical piece is built. Out of this also the sounds in the astral body appear. If you study the Chladni figures of sound, you will see that they have different forms, different rhythm and different tacts; otherwise they wouldn't be the result of music. We know that melody more or less belongs to the upper region; harmony and rhythm to the middle, the central part of our existence; and tact is more or less the expression of our limb system. And what today is jazz and all that, has drawn melody and rhythm entirely into the tact—and is nothing else but this [a bang on the table]. Whereas romanticism—well, I need not explain this.

MELOS	=	Gesture
RHYTHM – Harmony	=	Feeling
TACT	=	Character

But I must explain something else, and this is the following. Always trying to make it as easy as possible for the non-understanding people around him, Rudolf Steiner not only gave eurythmy (the art of movement), but he created the so-called eurythmy figures. You will have seen them: the I (EE sound) and the O—many other things. Not very pleasant to look at, but most interesting. And there he describes to the artist who has made these figures—and then to those who had great difficulties not to do but to understand eurythmy—he described it in the following way. He said you must see that the actual form of the movement, say the I (EE), is the gesture. But this gesture would be bare and naked if it wouldn't be filled with the feeling, with the emotion, and the emotion comes to expression in the veil which the eurythmist wears. And so in these figures you also have the veil in different colors and forms. You have the gesture, you have the veil. And then Rudolf Steiner describes a third thing. And this is what he calls (for lack of a better word, he himself says this) the 'character.' The character: whether for instance in one movement one part of the muscle is especially tight, or the forehead is wrinkled, or a foot is put down, so that it gives a very special accent to the whole figure of the movement. May I repeat. The gesture—say this—and the gesture permeated and shrouded in the veil of the feeling. And on to this, like the dot on the i, the character. And then Rudolf Steiner once also made the eurythmy figures for the minor and major scales, and in doing so he wrote: "Gesture is Melos, Feeling is Rhythm, Character is Tact." I am not telling you all this in order to bore you. I tell you all this because if we don't find the fundamentals, we don't understand the process of mathematics.

GESTURE	=	Melos
FEELING	=	Rhythm
CHARACTER	=	Tact

I go one step back and come then again to this. If I make a movement—for instance, I want to take the orange piece of chalk—I have done so ... put it back ... take it ... put it back. If you try to analyze such a movement, you will find it consists of gesture, feeling and character. It consists of Melos, Rhythm and Tact. Because the melody is stretched out between the motive 'I take,' between the vision 'here it is' (I must have taken it already invisibly), that means the melody of movement must have sounded through my sight, through my motive. Let us say: — U U, — U U. And now according to this melody I go there, harmoniously, rhythmically; and if I do it in tact, I get it. Can you under-stand this? Melody, harmony-rhythm, tact—and I've got it. If it's not an intentional movement but simply a gesture like the E (A), the melody is the gesture—I have this gesture: It is the one. It is here. This is the gesture or this is the gesture, or this is the gesture, or this is the gesture—whatever. It can be also this gesture: This is all melody. But how this melody is performed—whether it has harmony/feeling and even character/tact, or whether it is tact-less, has no character, bumps into everything and doesn't show any kind of harmony and feeling. If you make this conscious—and I try to do this in front of you—you begin to visualize, to realize, to experience what movement is. And we begin to experience this thousand-fold tone and sound within our astral body, which we with our tiny little ego, with our narrow consciousness, with our extremely limited conscious existence would never be able to govern. Don't think that you move; it's ridiculous. Our astral body moves, and however you try to hide your true intentions and movements, you can't; it's impossible. You can restrain yourself in giving someone else a kick, but he will know that you have to restrain yourself. It's quite impossible that the other one would not know that you are actually longing to give him a kick. Or if you want to appear as a very noble person, you will cut a ridiculous figure if you are not a noble person. Because the movements—this is 'I'—because it is our astrality. It's quite impossible to hide, whatever clothes you put on, one can see what kind of person you are.

Let us bring this more and more into our consciousness, to know: 'I am this melos, rhythm and tact.' And out of this, mathematics streams, and this is the only way to make objective and general what inwardly has to be so subjective and so individual. Can you follow what I mean?

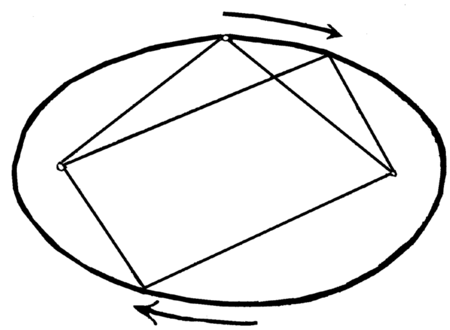
Now, whatever kind of mathematical process you perform is threefold. It is melos, it is rhythm, and it is tact. You can't help it. Whether you multiply or divide, whether you potentize or you extract roots, always you have the three. Either it is the sum or the product or what it is—it can also be the difference. And this, the end result, can also be the beginning wherewith you start, if you are a good teacher: this is the melos or the melody. This is what Rudolf Steiner indicates when he says, "Take a piece of paper and make many different bits out of it, or take a heap of beans or a basket of apples: This is the melody. And whether you, now in order to do an addition or a subtraction, start with the whole (with the melody) and part it, or put it together again—whether you teach the child '9 is 5 + 4' (and you start with the melos) or you teach the child '5 is 9 less 4' or you teach the child '5 + 4 is 9,' you alter melos, feeling and character (or tact or whatever you like). If you have division, for instance, say 36 divided by 9, this (see above) is the melody, this is the tact, and this is the rhythm. This is an indication—it should be worked out by you, and it can be worked out by you, because now it reveals itself: that every mathematical operation is in point of fact a piece of music. And this is why we can enjoy mathematics to such an extent.

36	÷	9	=	4
Melody		Rhythm		Tact

PRODUCT	36	=	9	x	4
SUM	9	=	4	+	5
DIFFERENCE	9	–	4	=	5

And Rudolf Steiner now, so to speak, underlines what I have told you. In a lecture on June 28, 1914, on the day when the Crown Prince Ferdinand was killed in Sarajevo, he speaks about the four fundamental curves: ellipse, hyperbola, the Cassini curve and the Apollonian circle. And he describes how the Four Rules—addition, subtraction, multiplication and division—are enclosed, enshrined, into these figures. For every mathematician this is a matter of course. But then he explains, in connection with architecture, the following. Perhaps I should make the ellipse in order to let you know what he means.

An ellipse has two focal points, and the circumference of this ellipse is nothing else but the line of constant sum-total of the two radii. Therefore you can quite easily construct an ellipse for children when you take two nails and a piece of string and now make this piece of string move around. It will be a most perfect and wonderful ellipse.



Can you understand what I mean? I only say this in order to explain the words of Rudolf Steiner, who says:

> *Da haben Sie das, dass Sie die Entfernung jedes Punktes von diesen zwei Punkten addieren koennen, und Sie bekommen immer dieselbe Laenge. So einfach ist das alles beim Kreis, dass wir nicht zu denken brauchen, nun aber muessen wir addieren. Beim Kreis sind die Linien alle Gleich bis zum Mittelpunkt, hier muessen wir erst den Gedanken haben, dass wir addieren. Nun koennen Sie selbstverstaendlich sagen: 'Ja, aber ich addiere gar nicht, wenn ich eine Ellipse sehe.' Sie nicht, aber der Astralleib, der addiert jetzt, und was der Geometer bewusst tut, das tut der Astralleib unbewusst. Er ist tatsaechlich ein fertiger Geometer. Und was Sie alles wissen im Astralleib, davon, vcrziehen Sie den Ausdruck, davon haben Sie keine Ahnung; da sind Sie ein ungeheuer weiser Geometer. …*

Now Rudolf Steiner says the following: If we look at such an ellipse, our astral body continually makes this addition of the two radii into a constant sum. This is continually done. And we know now who is doing it: this sound/movement, movement/sound figure of Melos, Harmony and Tact, which moves in our astral body.

Now this is what we have to regard first. These are the fundamentals of mathematics and human science. For us the question is—for you especially—how do we bring this all up in our children? So it is in the children, perhaps even in you. But how do we bring it up? How do we do it? There we would have to say the following. We discovered together yesterday afternoon that it is not the astral body in general which, when raised up, liberated, freed (after the second dentition), is the geometer, the mathematician. It is much more a very special part, a very special region of the astral body, the one which molds and forms the sense of life, of movement and of equilibrium in the first seven years of life. And I drew your attention yesterday already to this most important fact: how in the three lower sensory organs again some fundamentals of our mathematical processes are inserted. You remember we spoke about the three spheres which again are connected with the three parts of the human being: addition and subtraction (where there is difference), multiplication and division (where there is ratio, or we could say proportion), and then the logarithm. The question in the afternoon discussion was: How can we imagine it? The astral body is not yet born and is only born at puberty, but already it should do mathematical thinking. My answer was: Even in the womb, when the fetus grows, already from the third month onwards, it starts to twitch and then to move fingers, stretch and bend arms and legs, and so on. These are the first movements, and I say this in order that you have an image on which to rely when you teach. You must make the fetus of the astrality, still surrounded by the astral womb and sheath of the mother, mobile, make it move—with fingers, arms and even legs. Then you bring into activity what—out of the sense of life, out of the sense of movement, out of the sense of equilibrium—grows up.

Then there is a lecture which Rudolf Steiner gave on November 17, 1910, called “Mertschengeist und Tiergeist” (“Human Spirit and Animal Spirit”). In this lecture he tries to indicate from an entirely different point of view certain very special attitudes of the sense of movement as well as of the sense of equilibrium. And he puts the question: How is it, when the abilities of the sense of movement are enhanced, what would appear in later life? And if the abilities of the sense of equilibrium would be enhanced, what would then come to light? Let us try to understand this question. Rudolf Steiner describes the sense of movement, he describes the sense of equilibrium in very simple terms (it is a public lecture), he refers to the all-roundness of the then still ten sensory spheres, and tries to make his listeners understand that such a sense of movement is not something which is only a sense of movement. It is more. It has powers all over the human being.

The same holds good for the sense of equilibrium; the same holds good for the sense of life. But if we look now for this which is more, where do we find it? Here he doesn’t speak about the birth of mathematics after the seventh year. Here he speaks about a further enhancement, and he describes how the enhancement of the sense of life is the power of form. The modeling, for instance, of our individual skull, this is the last and higher part of the sense of life. And if we try to raise the sense of movement into a new sphere, them it turns into all that which is mimic: to do with one's face what the inner activity of the soul tries to bring to expression—joy, sadness, earnestness, sorrow, anger, wrath, intensity, thinking, everything. What then in our face rays up, from the whole of the sense of movement in our body, makes our face, our countenance, to be the image of our soul. This mimic, whether acted or natural, this is what intimately belongs to the sense of movement, what is its child. The plastic form, the mimic.

And if we ask the same of the sense of balance, we find all the gestures, the gestures either accompanying or not accompanying speech. Out of the equilibrium, out of this which I described yesterday as this immense manifoldness which governs, which keeps in balance all activities in our body, whether it be breathing or heartbeat, blood circulation or the building-up of blood, assimilation and destruction—out of this, all our gestures come. They describe the innermost equilibrium, in the language of movement. And with what is plastic, with what molds, with mimic, with gesture, you as teachers have the tools to enhance within the child the possibility to awaken or to create the faculty of counting (difference), because out of the sense of life there is freed the ability for addition and subtraction. Of mimic, because out of the sense of movement, there is freed the ability to behold ratio (proportion), multiplying, dividing. And from gesture come all the higher spheres of arithmetic: fractions, algebra, and so on. This, of course, is of the greatest importance for you to see.

You must only know that the dance, be it the folk-dance or anything else, is the true enhancement of all that which is our sense of balance and equilibrium. There you have it; and in the gestures of dancing—accompanied, inserted in sound, melody, whatever it may be—there you free all that which might turn into higher mathematics. In making the child consciously express emotions, you raise his ability to multiply and to divide. In making him consciously express forms, you help him with addition and subtraction—and this you can, of course, then do very individually. To learn this and to see how in all mathematics there is art—because this is, now, the art of dancing (whether you like it or not, it is an art)—the art of dancing, the art of mimic (which is drama to a great extent), a great plastic art, which then again takes into it mimic, when it portrays gesture and dance, when it tries to describe some attitude (whether preaching or whatever it is): All this is mathematics—all this is music; all this is permeated by melos and rhythm and tact. Because all this is astrality, taken into hand by the growing conscious mind of the individual.

This is how far I would like to come now. Only permit me still to give you a kind of 'bonbon.' In the same lecture of November 17, 1910, Rudolf Steiner refers to the man whom he met in his youth and who was the teacher of the poetess Della Grazie, Laurenz Muellner. And he points to a speech which Laurenz Muellner made in 1894, when he was Rector Magnificus of the University of Vienna. There he spoke in an amazingly enthusiastic way. He was then already an old man, rector of the Vienna University, and not even a Roman Catholic priest any more. He spoke then about the very strange fact that almost on the self-same day, on February 15, 1564 (almost exactly 400 years ago), did not only Michelangelo die, but also Galileo was born. And he points out (and this is the content of his speech), that Michelangelo, who created the cupola of Saint Peter's dome, did it out of his inner ability of being an artist, a sculptor, a painter, an architect, and then he died. And in his place (almost on the same day), Galileo entered into history, who then consciously found the laws of the pendulum and the laws of mechanics, out of which this was constructed.

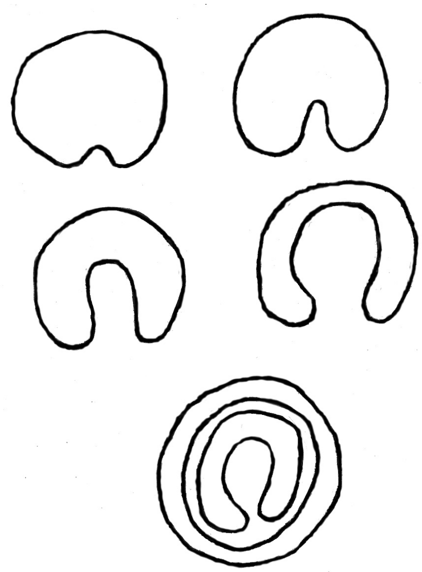
This afternoon we come to a kind of conclusion, with the actual conclusion tomorrow night, when I will try to deal with mathematics from a special point of view in connection with the development of mankind. So this afternoon I will try to emphasize only a few things which didn't strike home properly this morning. The first thing which is at my heart—that it shouldn't be left and forgotten but be taken up by those who have a greater understanding of mathematics and the teaching of mathematics than I profess to have—is the threefoldness. I have tried to point to the threefoldness in movement, the threefoldness in music, the threefoldness in eurythmy. We spoke of melos, of harmony/rhythm and of tact; we also described it in the words of Rudolf Steiner as gesture, feeling and character. When you take this over into the realm of arithmetic, you see that, in any kind of arithmetic or algebra, you must have three parts, $x + y = z$ is the archetypal formula. The whole of mathematics is enshrined into this: Here you have the three parts which otherwise appear as melos and rhythm and tact. To work this out would lead to further understanding.

If you read the way in which Rudolf Steiner treats the teaching of arithmetic (in the seminar course, or even in "Methodisch Didaktisches"), you will continually ask yourself why he divides if he explains multiplication, why he subtracts if he speaks about addition. It seems absolute nonsense! He starts from the sum-total, takes something away, and says that this is addition. He starts from the product and divides it, and says this is multiplication. If you don't just accept this because Rudolf Steiner said it, you may well be astonished about this. Why is it? It isn't just a question of the difference between multiplication and the way to teach multiplication. This would be as though I would set out to teach history, and I tell the children something about geography—because the teaching of history is geography. I only want you to understand it, and not to accept such things. Our question is: Why does Rudolf Steiner divide when he multiplies? Why does he subtract when he teaches addition? Within the frame of teaching, it is right that to teach multiplication, you have to divide; and to teach addition, you have to subtract. If you ask: Is this not connected with music? you would be right. This is what I'm after—because we said music is mathematics. And we can then go on to say that Rudolf Steiner wants the melody to appear first, before we really go in for the rhythm and tact. It makes no difference for a musical piece in which way you do it, as soon as you have the whole melody with you. Then there is no difference which way you put in the rhythm or the tact or anything. These three parts—the melody, the tact, the rhythm—they should be found in every possible counting, reckoning, algebra, whatever you do, so that you begin to experience:

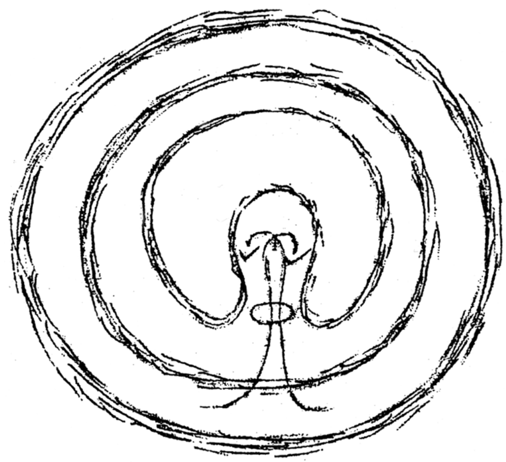
$$\begin{array}{ccc} x & + \ y & = z \\ \text{Tact} & \text{Rhythm} & \text{Melody} \end{array}$$

as a kind of song.

I would now like to say something which doesn't seem to be related to arithmetic, but you will soon understand why I speak about it. Wherever astrality works within an organic form—and you can say only there where astrality works—the form will no longer be a complete, closed form. I don't mean only a round form: It can be round like a flute, or oval like a flower, or elongated like a stem, or flat like a leaf—but it will always be completely full, completely closed. A root or a stone, a crystal, anything like this. But as soon as you come to the animal world, the completeness is gone; and instead of the completeness, holes appear within the form. Through these holes the surrounding powers—earth or water, light or whatever—stream into the organic form. The most primitive organic form is the blastula. Out of this most of the animals develop. In its earliest stages of development, the blastula is more and more indented. The astrality is at work. It is further indented, and still more indented, until out of this blastula a gastula forms out. This, new organic form has a hole—a hole which is mouth or anus or sensory organ. Into this hole streams the world, on the wings of astrality.



Dear friends, to see this and to meditate on it is most important. If you did this you would discover two things. As soon as the hole appears, movement comes about in what before was at rest. The outside within and the inside without create the twofoldness by which every such structure is built. The inside without gives rise to the development of the senses. The outside within gives rise to the development of movement. Sense perception on the one hand; movement on the other hand. And if this rises, develops, unfolds, if the senses create their own organs, if the holes become deeper, then a mouth develops, an anus develops, a lung develops. If the movement is not only along the body, then limbs develop in various ways. In this moment (and you know these are world-moments of tens of thousands of years), something has to start within the body in order to make it again into a unit, into a whole. Otherwise both the senses and the limbs would tear the body completely apart—the senses going out and coming in, the limbs going out and the powers of the world coming in.



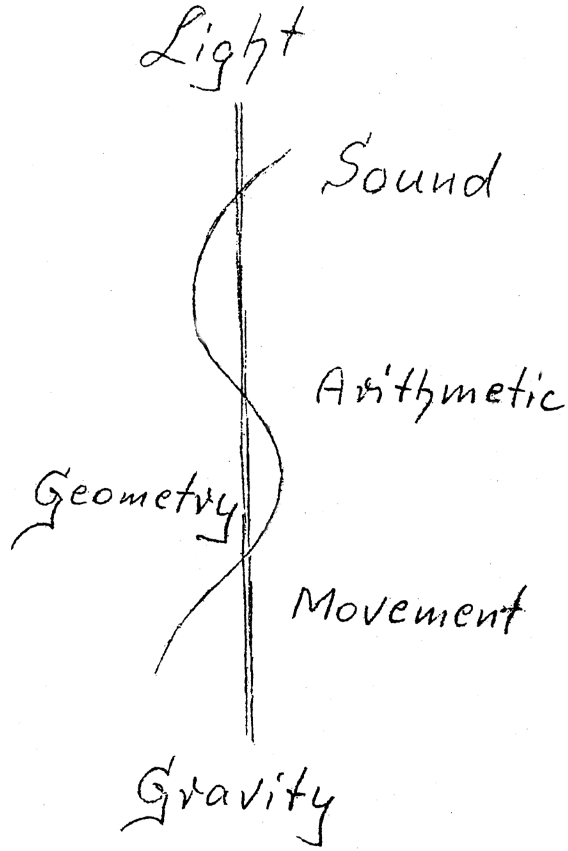
What makes it, what gives it to us, that we and the animals are not continually torn? Within the body something comes about again which is completely closed; it keeps the body as a unit, makes the body not open alone but closed in itself. It is the circulation of our blood which does this. The circulation of our blood is a completely closed circulation. There is no opening: wherever an opening appears, it is already closed—and if a hemorrhage comes about, we are ill. The blood clots simply to close the stream of circulation—not in order that we don't lose precious fluid, but to keep the oneness, the unity, of our existence alive. Now it is achieved: We have an astrality. We can move about, we can climb—everything is given to us. Yet at the same time, unification is achieved. A unification which keeps us at peace—and I use this word consciously—a unification which keeps us at peace, because only for this reason are we the same whether we stand here or there—or whether we get into an airplane and fly over to New York. We are in New York the same as we are here; in the morning when we wake up, we are the same as in the evening when we go to sleep: and this is due to the closed circulation.

You will now understand what I mean when I say that in this closed circulation there weaves, lives, acts, not only our ego; but there acts and lives within the sense of balance, the peace of our individuality. Here rests, is inserted, the living geometry, the staff of peace around which movement comes about. Without this, movement would not be possible. Children with loss of equilibrium can move quite freely when they sit down; but as soon as they have to negotiate the powers of gravity, it is impossible for them to move any more. In the lecture cycle which I quoted this morning (*Der Entstehungsmoment der Naturwissenschaft in der Weltgeschichte*), Rudolf Steiner speaks about movement and says:

Damals war es ein Menschliches, als der Mensch hinter seinen Bewegungen, hinter seinen Einordnungen seiner inneren Orientierung in den Raum, nicht nur innerlich noch erlebtes: Du gehst von hinten nach vorne und bewegst dich so, dass du dein Gleichgewicht erlebst von oben und unten, und du bildest vielleicht ein anderes Gleichgewicht mit dem Rechts-Links, sondern als der Mensch noch fuehlte, dass ja in jedem solchen Gehen, in jeder solcher Geometrie innerlich das Blut taetig ist. Es ist ja immer eine Blutstaetigkeit dabei,wenn ich nach vorne gehe. Und was war fuer eine Blutstaetigkeit vorhanden; als ich als Kind mich Aufrichtete aus der horizontalen Lage in die vertikale Lage! Hinter den Bewegungen des Menschen, hinter dem Erleben der Welt durch das Bewegen, das ja auch ein innerliches Erlebnis sein kann und es einmal war, hinter dem steht das Bluterlebnis. Denn in der kleinsten und in der groessten Bewegung, dies ich erlebe, indem ich sie selber ausfuehre, leigt ja das Bluterlebnis, das damit verknuepft ist. Nur, heute sehen wir eben das Blut als dasjenige an, was sich uns darbietet, wenn wir in die Haut stechen und dader rote Saft herausfliesst, oder wenn wir uns in aehnlicher Weise aeusserlich von dem Dasein des Blutes ueberzeugen.

This morning we dealt with movement. We saw that movement is identical with astrality, and that music is also identical with astrality. Now we go one step further. We look into the within of the movement. Rudolf Steiner tells us that behind—we should rather say within—every movement is the circulation of the blood. The circulation of the blood stands behind every movement, stands as the staff within every movement, because to make a free movement is only possible if within this movement blood is living, if within the movement the sense of balance creates a staff of peace and rest. Therefore the child is unable to move before he can stand, the movements are irregular, a kind of kicking. But when the child begins to stand and to make the first steps, he inserts himself more and more into the interplay between light from above and gravity from below. As soon as this is established, sound and movement begin to flow freely.

From here arithmetic stems, from here geometry (see figure opposite). All this in man, in the world, is one. In the normal person those different parts are a oneness. We try to analyze it only in order to see in the pathology, in the symptoms of inability, which of these parts is lacking. The living geometry powers are also earthly powers: they build the vehicle in which the ego rides.



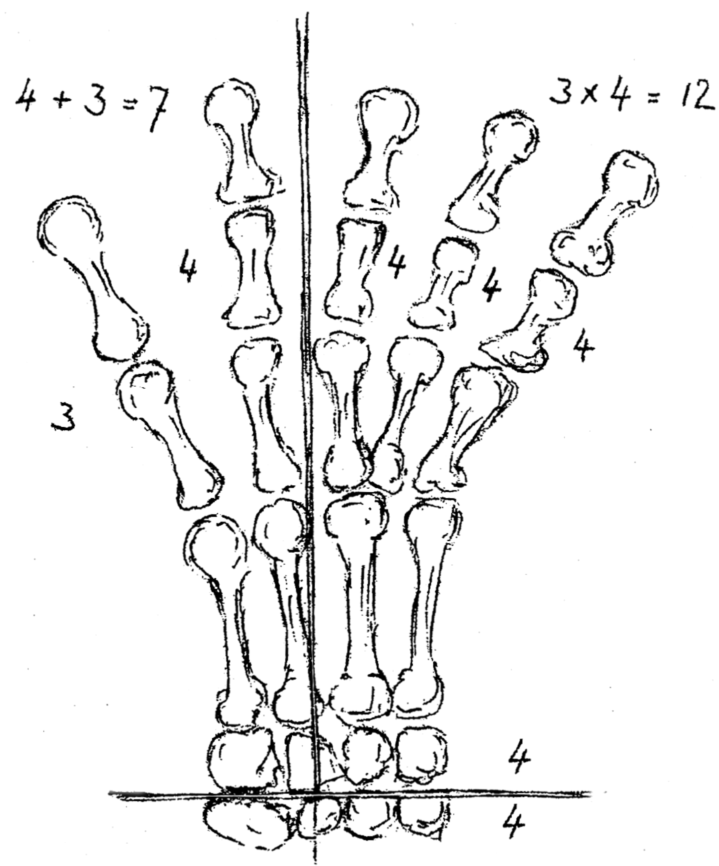
In this morning's discussion one of you brought the astonishing example of the girl who was not able to distinguish numbers, although she knows her letters. But the symbol of a number is not at all equal to the symbol of a letter, just as music is not the same as mathematics, and speech is not identical with arithmetic. Arithmetic is on an entirely different level of the soul. I will give you an example, an image. We make our movements by means of the muscles, and the muscles move freely for one reason: They have a staff around which they can move—that is the bones. All this is indeed limb; it is symbol. The bone is entirely different from the muscle: the one is movement, the other is speech, is vowels and consonants. Within this is a kind of Geruest, a fabric which holds it like the bone holds the muscles.

This image brings me to the very bold question: Where are the organs of mathematics in us? I have just told you it is the astrality and the sense of life and so on. But is there not something wherewith we continually do mathematics?

In a tribal society where no words or even signs exist for one, two, three, four, five, the people depend entirely on their fingers for counting and the simple forms of reckoning. A famous mathematician gave a serious lecture about 25 years ago: What, would have happened to mankind if by chance we had had six fingers instead of five? And he explained in exceedingly interesting mathematical terms how the whole of arithmetic and mathematics would have appeared completely different. This man was after something. He wanted to understand why we have five fingers, in terms of arithmetic—not because it's easier to hold things and so on.

In terms of arithmetic, why do we have five fingers? To let the children count on their fingers is absolutely justified. It is in line with the whole evolution of man. But how is all this built? Why do we count on the tips? You can say that the fingertip is the head part, then we have the middle system, and below the will part of it; and we count with our heads—it goes together with what we experience in counting ... difference ... sense of life. Or we can say that we have greater sensitivity in our fingertips. We have here, as it were, the head, with the bigger sensitivity. The bones are already sticking out above the muscles, and so the inside of the number is coming through. The network of numbers is appearing.

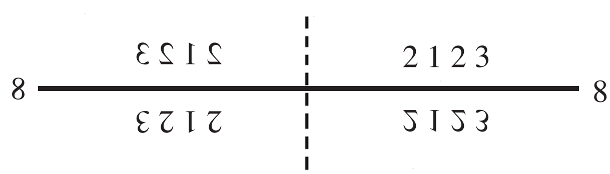
The anatomy of the fingers is quite amazing. There is a definite division anatomically, physiologically, between the thumb part (the radial part) and the other part (the ulnar part). If you count the number of bones in the thumb part, you will find 3 and 4—and these together are 7.



And if you count the bones in the other part, it is 3 times 4 which is 12. Here you have the whole foundation of the mathematics of Time and Space. You do it continually. You must only begin to understand it. And then we come one step further: to the eight parts of bones of the root of the hand, of the metacarpus. It is 4 + 4 or it is 2 x 4 or it is 23.

All this is 8. Everything is contained in there. It is made, it is formed out. And if you still go one step further, you then find the two bones: the radius and the ulna. This is now two. But it is not only two. In the hand and in the metacarpus, you have only two dimensions. It is flat. As soon as you come to the two bones (the ulna and the radius), you have already a third dimension. This is all inscribed; it is all here. Therefore Michelangelo could build Saint Peter's dome, because he had it in himself—and you have it too, and your children have it too. And only when we reach the top, we come to the oneness.

In the oneness of the humerus, everything is contained that forms itself out in the hand. The two which are separate have grown together, become one. The one parts into two, the two into eight, the eight into five, and so on. All this is mathematics, inscribed into our limbs. The limbs are the organs of movement; the limbs are the organ of music. But within these organs mathematics is present; it is alive. Imagine the following thing. If we look at our 12 pairs of ribs, the first one is small, and the next ones bigger and bigger and bigger up to the seventh pair. Then they grow smaller and smaller and smaller. Inscribed into this is the course of the sun throughout the year, growing ever more, coming back. Coming and going, The angle of this rib to the horizontal is exactly 23½ degrees—the same as the Ecliptic and the heaven's equator stand towards each other. Count the vertebrae and you will find that they are placed on top of each other like the days of the month: 28, 29, 30, 31 (it depends on how you count). The movements, the geometry, the mathematics of the whole world is inscribed into our skeleton. And as there are all the numbers, the forms formed out, no wonder that the muscles can obey all kinds of movement: because the skeleton is a keyboard—the skeleton is a lyre with so many strings that any kind of music/movement, movement/music can be played on it.



$$4 + 4 = 8$$

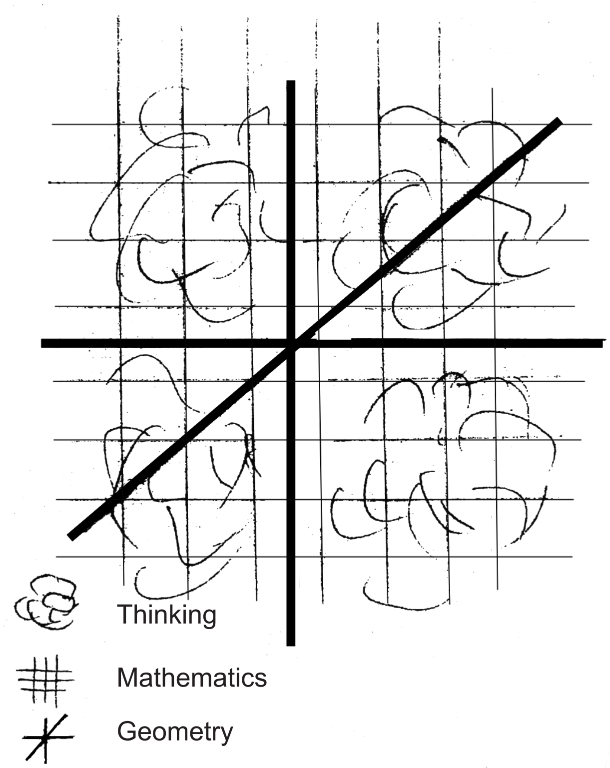
$$2 \times 4 = 8$$

$$2^3 = 8$$

And if we go still one step further, dear friends. We know how man is not living in one incarnation but goes from one life into the next—and how these organs, toes and fingers develop in one life and become our teeth in the following one. You remember how Rudolf Steiner pays such tremendous attention to the dentition. If we take this as the fundamental shape of our mouth (I am sorry to grin at you like this), our first teeth are 2, 1, 2, and they are also 2, 1, 2. Here again you have the whole mathematics permeated by the geometry, by the dimensions. And when you are seven you shed all this, and then still three more come. Then you have again the head—which is 4 + 4, which is 2 x 4, which is 2 raised to the power of 3.

On the ground of this you can develop anything. Here on your heads, and don't think I make a joke, there grow so many hairs continually that you can't count as high as the number of hairs there are. They provide you with any kind of number work. The whole of mathematics is inscribed into this. The teeth are the monument of mathematics. The bones are the living image of mathematics.

You will now understand my answer to the question of yesterday afternoon. This is thinking. Into this the pattern of mathematics is woven. And this is permeated by geometry. All three build our power of cognition. And the innermost—where our ego rests—is geometry. When you look at this image, you will understand why children should weave, should knit, should spin—if you want outwardly to develop what within the mind should come about. These are a few hints.



Reference:

Rudolf Steiner, *Der Entstehungsmoment der Naturwissenschaft in der Weltgeschichte*, Dornach, December 26, 1922.

It is a great joy now—at the end of the Teachers' Conference to be together with all of you, and to let you know what we have worked through during these days. The teachers have had the task to understand what it means to teach arithmetic, algebra and mathematics to the growing child. My task was to give a kind of foundation for the connection between mathematics and the human being, and I could do so because I don't understand anything about mathematics, and have therefore not lost my simplicity of thinking about it. We began to understand how man mathematizes—how it is that we as human beings have the ability and the possibility not only to count but to add and subtract, to multiply and divide, and to do even the higher forms of algebra and the highest forms of pure mathematics. This is a tremendous achievement for man—and the question was: From where does this start?

I will try to bring together a few of Rudolf Steiner's indications, through which we can gain a first idea of 'Man, the Mathematician.' When the little child begins to stand up to speak, to think, he is already forced to do living geometry and living mathematics. As soon as he starts to raise his head, he has to count, to measure between the powers of gravity and the powers of light or levity; and the process continues throughout the whole first year. When the child begins to move freely, he has to learn to experience the relations and proportions of the different parts of his body. And when he speaks, he has to learn to weigh up whether he speaks loudly or silently and low, whether he speaks quickly or slowly. All these are living experiences of man the mathematician. We have to learn to understand that mathematics is not what we can think. Mathematics is something which fills the whole human form, the whole human existence, the whole human life. And then when the child starts to think and to make the first conscious experiences of the world around him—when he distinguishes himself from the world, when he learns by trial and error to understand what is hot and cold, long and short, bigger or smaller, wider or narrower—these are not only experiences which he makes through his living body, but they are already implanted in his soul. This is the way we measure, number and weight (Mass, Zahl und Gewicht)—the three great powers of the master-builder of the World, of the supreme mason of the cosmos—have to be experienced. The young child learns to walk, and to speak and to think; but he also learns to insert himself livingly and with his understanding and feeling into measure and number and weight. By this he gradually grows to become a human being on earth. The human frame on earth—the earthly frame of man—is only achieved when measure, number and weight are conquered: livingly conquered first, then conquered by feeling and inner experience.

This comes to a certain first conclusion at the time when the second teeth are cut. Not only does the child become able at this moment to learn to read and to write, but he now also begins to be able to do arithmetic—to count, to add, subtract and so on. One of our achievements during this Conference was to become aware that to do mathematics (arithmetic) is something different from learning to read and to write. Rudolf Steiner has taught us how the living ether forces form, mold, build our organs and tissues during the first seven years and before, and how these etheric forces are the great sculptors of the inner side of our body up to about the seventh year. And at the moment when the second teeth begin to appear—when the edifice of the first period is put into our mouth—some of these etheric forces are set free and enable the child to grasp the world in the form of ideas. He becomes aware of written words and printed words, of letters and sentences, of the conscious formation of reading and writing. It is different with the powers of mathematics to grasp arithmetic—not livingly and by means of the inner experiences of the soul, but to discover the difference between 2 and 4, 6 and 8—and that 8 can be divided into 4 and 4, and that this is 2 times 4. To learn to think this comes about only after the seventh year, but it comes on a different level from those powers of thinking which make it possible to read and write.

If we ask: Where does it come from?, we can find an answer which Rudolf Steiner gives in a lecture-course in 1920 called "Grenzen der Naturerkenntnis" ("Boundaries of Natural Science"). He describes from where this power or this ability to do arithmetic comes. During the first seven years the powers of our soul—the astral forces, not the ether forces (not the powers of the etheric body but the powers of the astral body)—work within the three lower senses: in the sense of life, in the sense of movement, and in the sense of balance or equilibrium. There they help to build up the experiences which these senses have and give to the growing child. We have often spoken about what the senses of life and movement and balance actually are, and we should imagine that within the bodily etheric sphere of these three senses there works the astral body. What does this mean?

The astral body is a star-like body (I follow the description given by Steiner, though we must be clear it is not a body like a physical one). It is a body which is in continuous movement, going out and in at each breath, going out and in during day and night, going out after death, coming in after fertilization, building up the embryo. The astral body is as wide as the whole cosmos during our time between death and rebirth—reaching out to the stars, finding within itself the forms of the stars, the movements of the planets, the music of the whole cosmos, the geometry of the movements, the mathematics of the courses of the stars. Heavenly music, which is eternal mathematics, is inscribed into this weaving body of the astral, of the stars, and after birth this enters the body and undergoes 'earthification'; from being a heavenly existence it now becomes earth-like. And this earthfication means to live and experience within the sense of life, to live and experience within the sense of movement, to live and experience within the sense of balance. Imagine that the astral body is the one which builds the central nervous system in the embryo: the 12 nerves of the brain, the 12 cranial nerves—which is nothing else but the image of the 12 signs of the zodiac with the sun standing in the center. It builds the 28 nerves along the spinal cord—which is nothing else but the course of the moon around the earth. The astral body builds up such a form as our hand, where one thumb opposes four fingers—where the rhythm of one to four is the same as the rhythm of breath to the beat of the heart. All this living mathematics of the stars, which the astral body carries in it, has to be metamorphosed into the conditions of the earth.

The astral body enters the *sense of life* and in the sense of life it learns to experience a difference—the difference between the feeling of well-being and unwell-being. The child, who is the astral body, makes the experience of feeling all right, and then experiences the difference when it is hungry, thirsty, suffers pain, discomfort and so on. ... And then in the *sense of movement* the astral body learns to find out the ratio and relation of the limbs and parts of the body towards the stretching—all that is continuous, million-fold experience is an experience of learning the proportion here on earth. And in meeting the *sense of balance* the astral body learns a manifoldness which is hard to describe—a manifoldness which is not a simple experience of the negotiation between gravity and levity, between the darkness of earth and the light above.

SENSE of LIFE	Difference
SENSE of MOVEMENT	Ratio
SENSE of BALANCE	Logarithm

The sense of balance is the actual sense which rules deep within all the metabolic processes. The sense of balance keeps in balance the chemical equilibrium between base and acid; the sense of balance keeps in equilibrium the pressure of our blood, the velocity of our blood, the beating of our heart—the production of the billions of red blood and white blood corpuscles: All this right into the minutest thing, in a million-fold ways every second, is kept in balance. And here something is experienced by the astral body which can only be expressed mathematically by the word ‘logarithm’: it is actually ‘differentiation’ and ‘integration.’

We know that until the end of the second seven-year period the astral body will still be enshrined into the motherly astral womb; but nevertheless this part of it that has experienced difference, proportion and logarithm—in the depths of the sense of life, of the sense of movement, of the sense of equilibrium—this now rises up and. begins to grasp and to think what before has been experienced. And this mathematical, arithmetical-thinking weaves together and joins in with the ordinary thinking—with formulating of words, with judgment, with observation and all this. And so the child gradually builds up the beautiful pattern and net-work of its thinking ability.

DIFFERENCE	$8 - 2 = 6$	
	Add and subtract	
RATIO	$8 : 2 = 4$	
	Multiply and divide	
LOGARITHM	$8 = 2^3$	
	Raise to a higher power	
	Extract the root	

Difference, ratio and logarithm are the fundamental thought-experiences which we have to make in order to learn to add and subtract, to multiply and divide, raise to a higher power, and to extract the root (that means in German *Würzelziehen*). The child has to learn that the difference between 8 and 2 is 6, but the ratio between 8 and 2 is 4, and that 2 raised to the power of 3 is also 8. Try to understand what it means that with 8 minus 2, 6 is left. Or 8 and 2, 2, 2, 2 (4 times). Or 8 : 2 .. 2 .. 2. These are entirely different functions which we carry in our thinking. Professor Locher, the great Swiss mathematician (he was an anthroposophist), drew attention to the fact that *difference* (adding, subtracting) is intimately related to our life of THINKING; but multiplying and dividing (to judge *ratio*)—beautiful, more beautiful, most beautiful, small, smaller, smallest—this is much more experienced in the realm of FEELING; and here (therefore hardly anyone knows it). With *logarithm* we enter the realm of WILL, when we extract roots and raise to a higher power. This is the realm of numbers. This is something which it is so important to understand, and which we now gradually begin to find out.

The difference is experienced (I say this especially to the teachers and the therapists) between left and right, because here is the plane of thinking. And the ratio is especially experienced in up and down movement, because the plane of feeling is this horizontal one. And logarithm is especially experienced from behind to the front, or front to behind, because the plane of willing is the frontal one. If you bring all this together, you will learn to understand that *thinking*—all kinds and ways of thinking—is built up on this difference between left and right. In the animal there is no difference between the left and the right side of the brain—and only because there is this difference in man, thinking becomes possible. We learn to think in experiencing the differences between outside and inside. All this is the plane in the exchange between the left and the right, the day and the night side, the waking and the sleeping side of our existence. *Feeling*, however is a matter of proportion, of ratio. It is a matter of ratio between breath and heartbeat, between all the rhythms within our existence, the rhythms of sleeping and waking, between the proportions of the different parts of our warmth, air, water. If people who are suffering from jaundice suddenly lose all their inner emotions and feelings and become bored, this is only the result of the warmth proportion of the liver towards all the other organs having been lowered. The liver suddenly becomes (I exaggerate now) icy cold—it expands coldness throughout the whole body; the ratio of warm and cold is disrupted. Our feeling life is out of gear; antipathy and sympathy no longer meet in the right proportion. This all happens in the sphere between up and down. And *willing* rests, lives, acts, works continually in the never-ending building and destroying of millions of substances every minute in our body. Cells are created, cells are destroyed; blood corpuscles are created, blood corpuscles are destroyed; substances are born, substances die. This continuous metabolic process, which one can hardly describe in its immense manifoldness—this is willing. And continually it goes from back to front, and from the front to the back. These three together—*number* in our thinking, measure in our feeling, *weight* in our willing—build the mathematics, the geometry, the architecture of our bodily nature on earth.

After this first step we can ask ourselves: If all this happens in every single human being, how did it all come about in man and mankind—from the past to the present? How did it start? Rudolf Steiner was much concerned with this question. In a lecture on April 13, 1916, he describes the beginning of Atlantis, when man gradually became what he is on earth, a human being who is not yet endowed with but overshadowed by a personal ego, which gradually comes down. And he tells that together with this Menschwerdungs-prozess (this process of becoming man) the heavens were opened and man was endowed at the same time with the Uroffenbarung: with the Ur-old wisdom of man's connection with the universe, of man being a microcosm within the macrocosm, man made in the image of God. Through the events in Atlantis, this archetypal revelation of the holy heavenly Spirit gradually disappeared, and a last remnant was left to man.

> *Das Letzte, was der Mensch auf diese Weise gelernt hat, was von aussen ihm zugeflessen ist, das ist die Geometrie und Arithrmetik. Und derjenige, der heute noch Geometrie und Arithrmetik im wahren Sinne des Wortes auf sich wirken laesst, wird noch etwas verspueren davon, dass darin etwas anderes an ihn herankonmmt als anderes Wissen. Anderes Wissen Sammelt man so aus der Erfahrung zusammen. Aber Geometrie und Arithmetik ist etwas, worinnen man spuert, dass es wahr ist, abgesehen von der aeusseren Erfahrung, abgesehen von aller Sinneserfahrung.*[1]

Rudolf Steiner says the last which man learned in this way of the 'revelation of the great heavenly wisdom—the last thing which came towards him from without—is geometry and arithmetic. And if one today experiences geometry and arithmetic in a true sense, one will still feel that this knowledge is different from the ordinary knowledge, because we immediately know—without needing to prove it—'this is true.' The revelation of Atlantis gradually came to an end. It was still available in the post-Atlantean period right into the Greek time. Rudolf Steiner describes in this lecture how the Greek initiate was still able to create around him-self the etheric forms of the temples which he built—out of the inner mystery knowledge of mathematics and geometry. But then gradually it came to an end.

How, in point of fact, could this happen during the Atlantean period? Steiner describes it beautifully in the second lecture of the cycle on the "Wonders of the World and the Trials of the Soul." The physical body, becoming heavier, also drew the etheric body deeper into it, and by this something happened to the astral body:

> *…im menschlichen Astralleib wird die Fackel der Erkenntnis entzuendet, und diese Fackel der Erkenntnis aendert sich wiederum in Laufe des geschichtlichen Werdens der Menschheit: rische Bilderkenntnis hatte und heute die intellektuelle Verstandes-erkenntnis. So haben sich die Kraefte des Astralleibes geaendert.*[2]

It is in the astral body (so he describes it) that the torch of knowledge is enflamed, lighted—and this lighted torch begins with supersensible knowledge but gradually turns into intellectual knowledge on the path of mankind through the ages. The first ones who were guided by the great Manu out of Atlantis, from west to east—who were taught gradually to forget, to diminish the super-sensible knowledge of images, and to metamorphose it into intellectual grasp by means of ideas—the first ones who were capable of doing this, were the Ur-Semitic race of the Atlanteans. They prepared intellectually—prepared altogether—what later on became through them the civilization and culture of all the post-Atlantean civilizations. But this came about gradually, step by step. In India the Rishis still met with the great master-mason of the world, Zarathustra: he still knew that the 12 Arashaspands created the 12 cranial nerves, that the 28 Izards created the 28 nerves of the spine. The Egyptian and Babylonian men still knew the inside of mathematics; they were still able to a certain extent to raise the etheric forms of the temples which they built. This ability of master-masonry lived on right into the Middle Ages. The actual change came about almost exactly 400 years ago.

On the 15th February 1564 (in a fortnight it will be exactly 400 years—a very important day) the great Galileo was born. And three days later—on the 18th February of the same year 1564—the great Michelangelo died. And during these three days Michelangelo's building of the cupola of Saint Peter came to an end—and the man was born who found out the mechanical and isochronos laws of the pendulum, which made it possible to think this architecture which before was creative. You see, it's only four hundred years. And if you imagine that it is just 250 years since the higher forms of mathematics were thinkable—it started with Leibnitz and his time—you will more and more see what a tremendously long and extensive path mankind had to go. And yet in a mere 400 years there was built the whole knowledge of mathematics, in *thinking* of geometry, in *thinking*.

But we can still go back and look for a special point, for special people, for special destinies which have brought this about. Rudolf Steiner speaks of this in a lecture on November 9, 1909, in connection with the Gospel of Saint Matthew.

> *Es war gerade das hebraeische Volk dazu ausersehen, zunaechst eine solche Koerperlichkeit darzubieten, die bis in die feinsten Fasern des Gehirns hinein so organisiert, dass das, was wir Erkenntnis der Welt nennen, ohne den Einfluss des alten Hellsehens zustande kam. Das sollte die Mission dieses Volkes sein. In dem Stammvater dieses Volkes, in Abraham, war auch tatsaechlich eine solche Individualitaet auserlesen, dass dessen Leiblichkeit ein geeignetes Instrument war fuer das urteilende Denken. Alles was vorher gross und bedeutend war, stand noch unter den Nachwirkungen Alten Hellsehens. Nunsollte aber eine Persoenlichkeit ausersehen werden, welche das geeignetste Gehirn hatte, um sich nicht draengen und stossen zu lassen von den hellseherischen Imaginationen und Intuitionen, sondern berufen war, die Dinge rein mit dem Verstande zu betrachten … Waehrend in der aegyptischen sowohl wie in der chaldaeisch-babylonischen Kultur noch die Nachklaenge des alten daemmerhaften Hellsehens da waren, wurde aus dem chaldaeischen Wolke ein Individuum auserlesen, welches nicht mehr darauf aufbaute, sondern auf die Beobachtungen der Erscheinungen der Aussenwelt. Damit sollte jene Kultur eingeleitet werden, deren Fruechte noch heute unserer ganzen westlichen Kultur und Zivilisation einverleibt sind. Jenes kombinatorische Denken, die mathematische Logik wurde durch Abraham eingeleitet; ihn sah man bis ins Mittelalter hinein in gewissem Sinne als Vertreter der Arithmetik an.*[3]

We should remember what I spoke of earlier—about those who came over after Atlantis to the West, bringing the capabilities for starting the Post-Atlantean culture, which is to lead from supersensible knowledge to intellectual knowledge. This is the knowledge which, within the astral body, turns the world of images into the world of ideas and concepts. Here the Ur-Semitic people showed the way. And now in Chaldea, roughly two thousand years before Christ, a man named Abraham was appointed. Abraham had, on account of the special formation of his brain and body, the possibility to introduce the image-free civilization of the Jewish people. This is one of the great missionary tasks of the Jewish people: to introduce mathematical image-free thinking into the world. This did not develop in everyone; it had to be carried by a very special race—by the Jewish people, by the Hebrews. Abraham was the first who was chosen and by the gradual preparation of the whole of the people, by the selection of the children, by the selection of men and women, this brain formation more and more became the possibility to think in mathematical and arithmetical terms.

It went on: Moses built a tabernacle, an expression of mathematical architecture; King Solomon built the temple—an expression of mathematical architecture. And in the end, on the Hill of Golgotha, stood the Cross. This you must learn to under-stand; this is one way from Abraham to Moses, to Solomon, to Jesus and the Cross of Golgotha. If we begin to see this, we will understand that at the turning-point of time (two thousand years ago) not only were two children born—but also two streams of knowledge came to an end: the stream of the Kings—the end of Mathematics—which was still alive, still inserted into the heavens; and the stream of the Shepherds the knowledge of the heart. All this is one great process. The sea of knowledge swept on further went through the next centuries, but the waves became lower and lower. The temples, the churches, the cathedrals, the graves, the monuments became less and less spectacular— less and less true, until there arose the great physicists, the great mathematicians, the learned men.

Where do we stand today? If we look out into the world, everything is bare mathematics. The musicians compose on the writing-desk, with rules and pencils, multiplying, dividing notes, jumping with them. Sculptures are nothing but lines and holes and circles, with no life left in them. Look at the painters, look at the physicists: bareness, poverty, emptiness. The skeleton of mathematics is left—but it is almost a rotting skeleton.

All this is the result of the complete end of that wisdom which was given to man at the beginning of the Atlantean period. When Steiner spoke about it at Christmas 1920, he formulated a poem which I would like to read to you.

Isis-Sophia,
Des Gottes Weisheit,
Sie hat Lucifer getoetet
Und auf der Weltenkraefte schwingen
In Raumesweiten fortgetragen.

Christus-Wollen,
In Menschen Wirkend,
Es wird Lucifer entreissen

Und auf des Geisteswissens-Booten
In Menschemseelen auferwecken,
Isis-Sophia,
Des Gottes Weisheit.

Isis-Sophia,
Wisdom of God,

Lucifer has slain her
And, on the wings
Of the world-wide forces,
Carried her hence
Into cosmic space.

Christ-Will,
Working in man,
Shall wrest from Lucifer

And, on the sails
Of Spirit-Knowledge,
Call to new life
In souls of men

Isis-Sophia,
Wisdom of God.

This is the true story of mathematics and geometry. The Ur-wisdom was taken away; what remained in man is today shadowed, tattered, torn and rotten. When Rudolf Steiner described the disappearing knowledge of the Kings and what it should become today, he said it has to be transformed into Imagination. That means that Mathematics, Arithmetic, has to become Image again—has to be seen, has to be experienced. We have to learn to understand how Difference, Ratio and Logarithm work in us, and that this (within and without) is one and the same.

I hope I have brought you a glimpse of the mood which we should engender when turning to the true mathematics. There is a moving story in Rudolf Steiner's Story of His Life when he describes how as a child, when meeting geometry for the first time, he experienced true happiness and joy. Some of us can understand this; those who don't should try to do so.

References:

1. Rudolf Steiner, *Gegenwaertiges und Vergangenes im Menschengeiste: Die Uroffenbarung der Menschheit*, Berlin, April 13, 1916.
2. Rudolf Steiner, *Weltenwunder, Seelenpruefungen und Geistes: Offenbarungen*, München, August 19, 1911.
3. Rudolf Steiner, *Die tieferen Geheimnisse des Menschheitswerdens im Lichte des Matthaus-Evangelium*, November 9, 1909.

I.R.: This morning it was uttered by so many that they feel even if a child would not at all make use of Arithmetic, it would be necessary to introduce it in main lesson, in order that the fullness of the humanity would be cared for. It came up just as a teachers' impulse, and I found it so wonderful that now the answer was really given. Just as we can't withhold high moral ideas from a child because he or she can't 'make use of it,' just as little can we withhold this which has so much to do with the substance of our existence. We all have probably occupied ourselves with such words by Rudolf Steiner that Arithmetic stands in the middle between Art and Reading-and-Writing, which perhaps has not yet had a real recognition: one accepted it, but one didn't quite know. Still we say "the three Rs" as if the third R belonged to the two Rs, and it actually does not.

Dr. König: For me it was so important—and I think it will be more and more important for you all—to realize that mathematics is astrality, is the stars. This is so important. It is music. It is movement: it is not etheric powers.

I.R.: When did arithmetic come about?

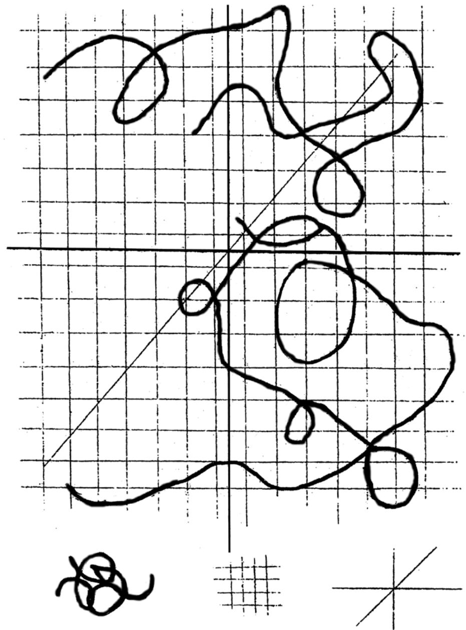
Dr. König: Well, Rudolf Steiner says very much more. There is a lecture cycle of three lectures—it's in connection with the Gospel of Saint Matthew—in which he describes Abraham as the first mathematician: he brings mathematics into the world. Perhaps I will read a few sentences of that to you tomorrow. It's Abraham.

H.C.V.: Is that not in connection with thinking without pictures?

Dr. König: In connection with thinking without pictures—yes. But he calls him exactly "the first mathematician." It's also described in the first volume of the history by Bock.

F.H.: There was something in your talk just now which in a way puzzles me. It describes there the thinking in two different ways. You described the ordinary thinking of everyday life and then the mathematical thinking. If you could elaborate on that—with regard to that the one is the observer or the looker-on of the other one.

Dr. König: No, not the onlooker of the other one. But mathematics, so to speak, gives the kind of 'Raster' pattern: it establishes the pattern within the whole sphere of thinking. The thinking is a kind of weaving. There is a drawing which I wanted to show you. If we imagine this as the fabric of thinking, we would have mathematics as a kind of pattern going through it: you can also call it a warp. And geometry is this. Geometry is still higher than mathematics. Mathematics is higher than thinking. In your thinking you can think very well without being a mathematician—but mathematics holds the thinking in patterns, and the structure (the innermost structure) is given through geometry.



I.R.: If we then have to think of this remark of Rudolf Steiner's that his first experience of happiness was geometry, then one would feel in your drawing that geometry is nearer the ether world, where our thinking is imprinted. "Unsere Gedanken sind schattenhafte Abbilder"—this to be a shadow of the etheric holds much more for our ordinary thinking than it holds good for mathematics, and geometry is most active in the realm where thinking originates—or is at home.

Dr. König: I would perhaps formulate it so that I say the ordinary thinking is in the world of shadows. Mathematics is the fixed points of light like the stars: It is more or less astrality. Geometry is only possible if we have an ego: and the insertion of our ego into thinking—this is geometry. The animals have no geometry; therefore they walk on four legs. But we must establish our own geometry first through the sense of balance—and in this way only man has a sense of balance. I know all animals have a sense of balance, but man has the sense of balance that gives him the ability to geometrize—and this is the ego.

U.S.: I have a question and that is: 'As thinking arithmetic and geometry follow each other, is it so that geometry could not possibly come about if there is too little arithmetic being done?' That means if one would always stick to the lowest possible forms—addition and subtraction—that certain things would not come about?

Dr. König: This would not always be like this, because they are also independent—and you could have a child, say, who has a very strong sense of ego and yet his sense of thought is underdeveloped. In the same way you can have somebody who has a very special ability for geometry yet not for mathematics.

U.S.: I have such a boy. My other question was whether one could help arithmetic in doing geometry.

Dr. König: I am quite certain—but we are going to speak extensively on what we can do in order to raise the abilities of number-work. Because there I really think, dear friends, what I indicated at the beginning—to make it either visual or audible or touchable. This is the question I wanted to take up. I don't think it is by mobility—because if you move you push the sense of mathematics back into the lower senses; and our question is to raise it up. You, of course, make it available: I wouldn't say: 'Don't do anything with movement.' You can do it. You make it available for the child. But if you want to teach it, it is something entirely different.

K.v.S: What do you mean when you say: 'The third possibility is through touch'?

Dr. König: That you, for instance, clap—or that you teach a child to distinguish between the end, the middle and the lower phalanx of the fingers. They should learn to experience this. It is something entirely different. And thereby arithmetic ideas of difference rise up.

K.v.S.: Is it connected with numbers?

Dr. König: It can also be connected with numbers. Why not? One, two, three, four, five [touching each finger in counting]. Is this not an experience of touch? Do you think a child would be able to do this in looking at the fingers? Or you can make it 'one, two, three, four, five' [moving each finger]. That means sense of movement and sense of sight. This is so important. Don't overlook what touch means: one, two, three, four [stamping on floor]. This is touch. It is touch, it is sense of movement, and it is sense of hearing.

K.v.S.: I have always seen this very much in the sphere of time and sound rather than of touch.

Dr. König: But how can you compare time and touch? The latter is a sensory experience, while time is something entirely different.

F.B.: We do not make use of movement, of touch and equilibrium in introducing certain number-work and in preparing the child (especially our children) help their incarnation process catch up, so that then the possibility to do number-work arises.

Dr. König: One can also do this, of course.

F.B.: The chaos of movement which is present in the very young child is through the uprightness first enhanced to speech and then to consciousness. When we make use of touch or balance, we are somehow going back in the child's development, so that we can then introduce arithmetic. This morning it was obvious that it is not arithmetic when we count by stamping. This is only individual teaching, coaching.

Dr. König: This is what I tried to describe with the Platonic ideas, the latent possibility in everyone to do arithmetic.

E.v.d.S.: There was a boy who was quiet and very observant but incapable of counting more than two objects. But we found he could not really get himself off the ground. He would shoot about much faster than his legs could carry him. It was impossible for him to hop, for instance. Then, after two terms, hopping was done daily, together with speech exercises. He had to try it, and from the day when he first managed to hop, he could suddenly begin to count. It went then really much better—but really from that very day onwards.

Dr. König: Certainly—because in that moment he learned to rule, to a certain extent, his sense of movement, and thereby forces were set free which could count. That is quite clear.

E.v.d.S.: There are children who are not able to walk One-Two-Three. They always walk four or five steps or only two steps. And I think that is something which has to be conquered…

Dr. König: But this is the preparation for number-work and not the teaching of number-work. And this we must distinguish very clearly. You will have to make many, many, many exercises and movements and rhythms in order to free the astrality from the sense of movement, sense of balance—because otherwise number-work will not be possible. But to teach number-work is something else from the preparing to teach number-work. This you must learn.

K.v.S.: I would like to come back once more to this great realization connected with the step that you say the child has begun or accomplished with his second dentition—that then astral forces are at work and not etheric. Because I think there has been a lot of unclarity about that. And this step in itself is such a mystery that we often use it and talk about it—but I don't know whether we really understand what it is. And I wonder whether you could possibly say something still to it.

Dr. König: Well, I've tried to say a great deal about it already. I mean, I've tried to say how the powers of the soul work in the sense of life, in the sense of movement, and in the sense of equilibrium—and that the same things which they establish in bringing about the experiences of these three senses is lifted up, and difference, relation and logarithm are there.

K.v.S.: But this is lifting up.

Dr. König: It is freed. If you stir, for instance, a pot in the kitchen, and then you use your hand to make a drawing of the action of stirring, it is lifted up. I mean you need not visualize this. Can't you see that at one time the hand is doing one thing, at another time something else?—so soul forces at one time are working in the sense; then the bread of the senses is baked—you can take it out and use your hand for something else.

I.R.: Is not the difficulty this: that we so often talk about the freeing of the etheric forces with the seventh year. Now we know that the astral forces are still fettered. There lies this mysterious thing that, in the course of the next seven years, frees itself, and we also don't teach logarithm before that is.

Dr. König: Exactly—but these parts which work in the lower senses are already freed at the seventh year. They are here, you see—they are liberated. The others are still working. Because also the astral body is, of course, much more complicated than the physical one. Already the etheric one is so complicated—not to speak of the astral one. It is tremendous. And what holds good for one part doesn't hold good for the other. There are enormous powers still enshrined in the motherly sheath—enshrined in the tissues. But those working in the lower sense, there you have this little bit which you can use for adding, subtracting and so on.

H.C.V.: I think it throws tremendous light on the curriculum of the First Class, where Dr. Steiner teaches that after one has achieved with this very special introduction of the first of the Four Rules the first glimpse of an understanding, then the child has a duty to learn it by heart. Then it goes down into the etheric thinking—and not before.

Dr. König: Exactly—and imagine the point: that as soon as possible one should start with the rules and learn them by heart: because then you insert it really into the etheric body. And he makes also a point to learn all these four at the same time. I find this most important: that one should not go from addition to subtraction and then wait—and then perhaps go to multiplication. Do it together—because it is one.

H.C.V.: But it shows also that one probably should not ask a child to learn something by heart before this first glimpse in the astrality thinking is there.

Dr. König: Exactly. The star must shine—and then it can mean something.

F.B.: This has, of course, been sometimes a painful experience when we have tried to let the children memorize their multiplication tables and there had been no glimpse before that. It was a waste of time.

Dr. König: Not even a waste of time. It was destructive for the child.

F.B.: Also what was described this morning as experience in other circles than ours. Of course one knows that multiplication and division are manipulations which are not really common to daily life and are therefore also not so necessary, and one could just as well leave them away—certainly division. But to come to a grasp of these four manipulations I think is very, very important to begin with.

Dr. König: Fundamental.

U.S.: I think there are always two views to it. One is perhaps wondering what I am doing with the child or to the child in introducing or in practicing, and the other thing is (much later): 'How long shall I go on, and what does the child take of it for life?' That is a secondary question, actually, to the first one: 'What am I really doing?' And we must be quite clear what is happening within a child in doing it and doing it rightfully.

I wanted to mention to you also: We have had the good luck to make the acquaintance of the headmaster of an E.S.N. school, who is a wonderful person and who has given us two lectures. Now one can see that this man—with his philosophy and his psychology—stands firmly on his feet. But I had all the time the feeling we must also be safely on our feet too—in order to answer his questions properly.

Dr. König: You think he has questions?

U.S.: Oh, yes, he has questions.

Dr. König: Good.

U.S.: In the E.S.N. schools they teach the children individually, and he also wants to bring about some kind of class teaching. This is his question, how to do it. And I feel if we could give him an answer that would satisfy him, he would then try to introduce such things.

H.v.W.: What arises so very much out of this is that with arithmetic one needs patience through the years, and indeed an inner certainty if one really also wants to practice that one should not train before this bridge of understanding is there. Because the outer side works against it. One can achieve in another way a certain measure of results outwardly.

M.T.: In connection with the freeing of the astral body which makes it possible that logarithms come about at the fourteenth year, you said this afternoon that the ability to start arithmetic begins at age 7 with the change of teeth.

Dr. König: Yes—and then it gradually widens. You see, you must see logarithm as the end of a development: over difference, relation, logarithm. And there you can go, of course—7, 8, difference; and then soon relation—so that difference and relation are established by the ninth year. And then wait. Some will have it earlier. If you imagine normal children today, they will extract the root and raise to a certain power; they can do it at 10, 11 already. I mean they get the idea of it. Everything else is practice.

I.R.: And has this earlier development to do with an earlier freeing also of the lower part of the etheric body?

Dr. König: No doubt. This is one of the roots of acceleration. The cart is gripped; the wheel is rolling much quicker.

M.T.: Are some astral forces available at the age of seven?

Dr. König: Yes, *these* astral forces—not all. There are different astral forces. You couldn't say that all etheric forces are born at the second dentition. We wouldn't be able to live up till then if all would be enshrined in the mother's womb. It wouldn't work. Before birth the child has been developing for 280 days, and also hundreds of years in the spiritual world—but at the moment of birth it is born. So also the etheric body develops and then is born—and when it develops many things are freed. The little child in the womb starts to kick and then to move, to swallow, to digest, everything. You must take such things as the final moment. The birth of the etheric body is a long process.

M.M.: You prepare also the second teeth already long before they come out. Therefore why should you not prepare the astral quality of arithmetic already long before it comes to fulfillment?

Dr. König: Exactly. In the embryo you can count already the first teeth, and soon after birth all the second ones.

H.C.V.: I wonder whether one can see in these mathematical astral forces also the soul tendency to analyze—and more in the underground of etheric thinking forces the tendency to synthesize.

Dr. König: Yes, in one way this is quite true. You refer to this one very important lecture, in the Basler Course, I think it is. I am still struggling with what he actually means, but it's a possibility what you said. I'm not quite certain, because he connects it with freedom. I wonder very much.

H.C.V.: But it would also throw a light on the way of teaching the Four Rules, when one starts in the analytical way, involving these quite newly-born soul forces, and then (in learning by heart) one builds on the other forces and goes back to the ordinary way of teaching.

Dr. König: It is certainly a possibility to understand it—definitely. But whether it is everything he means is a great question to me. Because he so exquisitely speaks of the soul forces of analyzing as well as synthesizing, and this is my question.

U.S.: I had still a question to the layout of arithmetic during the year: not so much the second lessons—it's quite clear that they must be continued. But also in the main lesson an introduction into something new wouldn't be enough, and one should rather also have it each term, even if it is only for one week. What would you think about it?

Dr. König: I think, in the realm of number-work, repetition is something of the greatest importance. It's also in the curriculum that one should continually repeat. Because, you see, the mathematical memory is something entirely different from the visual memory. In point of fact you can't remember mathematical things. I'm sorry to say this, but I know it. It's exactly the same as you don't remember any kind of spiritual experience: it's impossible. It's lost, and you have to revive it again—to redo it—in order to get it. And so it is with mathematics, because it's a realm which has to be conquered and always conquered anew. And if you remember seven times six is forty-two, this is not memory—this is repetition.

E.v.d.S.: It is a kind of habit.

Dr. König: Yes, you can definitely say it's a habit. I would fully agree. You know it exactly as you know how to drive a car or to ride a bicycle or anything else.

M.M.: But why does one then say one wants to ingrain a multiplication table into the etheric memory?

Dr. König: I wouldn't say into the etheric memory—into the etheric body. And to ingrain it means first to inscribe it into one's memory and then to have it as a habit. When you learn, for instance, to swim, you must first remember how to do it, before it is done—or to drive a car, or so.

U.S.: Now the question is where to limit oneself. For instance, with the multiplication tables, I would think with a class which has great difficulty with the tables, one should not attempt to go too high. I would also not think that one shall limit it to tables of 3 or 4.

H.v.W.: May I ask still a question in regard to what was said more as a general indication towards pre-number-work? Because if one now imagines that one has a 7- or an 8-year-old child, and he doesn't get his first glimpse of understanding, then one should start then again with exercises. And there was this wonderful example of Elisabeth's about this child. Now we are in the situation that one can also have a

14-year-old child who still hasn't the first glimpse of understanding. His whole constitution is then, of course, totally different—because he is then a 14-year-old and also his astrality has developed fully. I wonder very much if one could see also here in general some indications in what field, in what realm one should again introduce pre-number-work. Because I find this in the meeting with such children always a great difficulty.

Dr. König: It will, of course, very much depend on the individual child and I think a general answer would not be possible. I don't think so. Because then one would have to find the exact remedial exercise in order to make this possible. It may be playing the flute or recorder—or something like this, or making a very special speech exercise, in order not to engage the whole motor-body but a special one. So I would see it.

N.J.: I had the experience with a backward boy who couldn't count at all—and it suddenly came after he learned to knit.

Dr. König: Yes, certainly.

N.J.: It was quite indisputable that it was as a result of knitting that he could count.

Dr. König: I remember a boy who was quite unable to grasp multiplication and division (it's long, long ago—30 years) and we taught him Latin grammar, and in this moment he got it. Within a few weeks he was able to do it. Because the Latin grammar shaped his logic—and then it was there. But don't use such an example, please.

F.B.: I have a very simple question—that is, concerning the acceleration—because that is a phenomenon that there is this acceleration. It's maybe not so pronounced in Britain, but also there. And yet we can read from it that a certain process has come to a certain conclusion. Of this the change of teeth or puberty is an expression—and yet, if that has really taken place earlier, what are we to do in school? Take the child earlier or not? That is my question.

Dr. König: No. You see, I have the impression that for the acceleration, which is a disease of mankind, we have to use the remedy, and this is the curriculum. Even if the individual child suffers under it, it should serve mankind, it should serve the whole—in order that there some remedial powers are introduced.

H.G.V.: Has one not here to distinguish between class-teaching (where one has definitely not to regard any abnormality) and remedial exercises for the individual pupil?

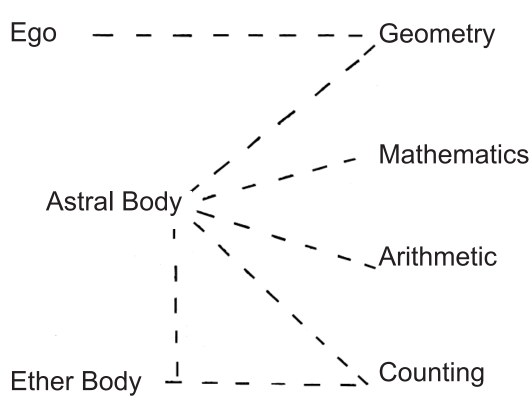
Dr. König: On, definitely, definitely. I only speak about class-teaching now, because for something else you will have to use many different things. But the curriculum is class teaching.

M.K.: You mentioned yesterday that geometry would have to do with the special quality of the ego, and you said that the astral body unconsciously performs geometry in our body all the time. I wondered how that goes together: if perhaps the ego does it consciously.

Dr. König: The ego does it also unconsciously, but if one says the astral body is doing something—if, for instance, Meta is a teacher—it doesn't mean that somebody else is not also a teacher. Also the ego—and especially the ego—inserts itself into geometry. Can you please understand that it is exceedingly difficult to distinguish in practice the deeds and sufferings of the astral body and the ego. What the astral body does is, of course, followed by the ego; what the ego does is, of course, followed by the astral body. Why must you make such decisions?—that the ego lives on the second floor, the astral body on the first floor, the ether body on the ground floor and the physical body in the basement.

M.K.: Because yesterday you bound it up with the ego.

Dr. König: I didn't bind it up; I indicated. You can also do mathematics consciously, and then, of course, your ego is involved. You can't help it. Astral body (and now I refer to the various floors of your supersensible house), Ether body, ego—Counting, Arithmetic, Mathematics, Geometry.

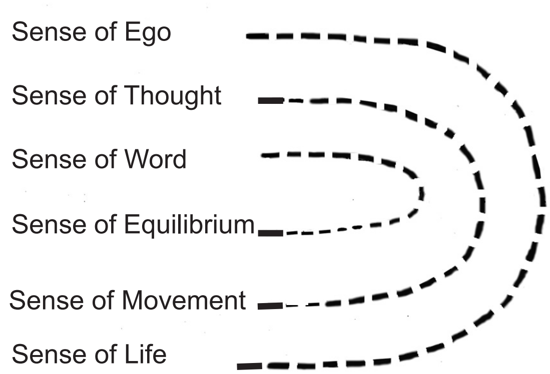


U.S.: I have a big question: I don't know where it leads to now. That is if one follows up certain age-groups, and always something special belongs to it. Let me say the tables belong to a certain age, if possible before the ninth year—up to the ninth year. If they are not learned then, it's gone—and you can do what you like, they can't get it by heart so well any more. Now also other things in arithmetic belong to certain age-groups. Can one understand a little bit more about the difficulties children have to get a grip: because if we are imbedded in arithmetic, then actually it should be quite near to us. I understand that the difficulties are always different whatever the child's handicap. But I must say there is one thing I did not quite understand. That is in looking at these astral forces of certain senses, it should actually be so that there are children who might have possibilities with addition—that is quite clear. They are able to do addition but not multiplication. But why is it so that these steps are really built on each other? It could also be so that some child has great difficulty with addition but nevertheless would grasp multiplication.

Dr. König: Which you find. Yes, of course.

U.S.: You see that would justify that one really does it at this certain stage, nonetheless.

A.P.: But how can that be if you cannot count? How can you then get the idea of multiplication?



Dr. König: Don't you know that many children, for instance, are able by thought to calculate the most amazing sums and product. If you ask them: 'How much is 364 times 5280?'—in a flash they would tell. Have you never met this? And sometimes such children are unable to count, to do simple addition. Because there something already has developed. I give you another key. I would have thought you know it already. The sense of Life, the sense of Movement, the sense of Equilibrium—they are connected with three higher senses: with the sense of ego, the sense of Thought, and with the sense of Word. These, these and these. If you can see this, you will, for instance, see how suddenly the sense of Thought is already developed—which in the sense of Word has not come about, and vice versa. This is quite possible. You can have children who have not learned to speak, but who are able to think. You have children who are not able to speak but who have a very present ego. And such things you must not only learn to observe, but out of the observation to use as curative exercises and qualities. This is necessary.

U.S.: Connected with this is another question. How is it if always new children stream into the class? Shall one give a short repetition—as one does in history or geography, if there are so many new children? Or shall one just carry on with what is necessary for that age?

Dr. König: I wouldn't be able to say this clearly. I've never had a class. I wonder what the teachers would say: They should answer you. I'm sure that you have experienced this many a time. I would be very interested to hear what you think about it.

F.B.: I would think that is a question which one can't answer in general because it depends on what we refer to: whether we refer to the subject-matter of the curriculum or to the children's ability. If you refer to the children's ability to do arithmetic, then I think group-work is anyway something which we should not attempt to be a uniform activity, but something which is an individual activity, possibly within a group—and then this question would not arise.

U.S.: No, the curriculum.

F.B.: In the curriculum we always go ahead.

U.S.: Is it justified to develop the arithmetic curriculum if the fundamentals haven't been grasped in one's class?

Dr. König: But why can't you then take such a child separately for two or three hours and tell them about it? As you have so much fantasy—teacher fantasy—to do this. It's an unjustified question, actually, because you should find your own answer.

U.S.: That is all right if there are only one or two or so children coming newly—but we had, for instance, now a great change, and I think it happens now that suddenly ten children of different stages suddenly entered.

Dr. König: And most of them have come without previous schooling?

E.v.d.S.: Some with, some with none. And if in one class there come some with and some without—and they're twelve years old—it's quite a problem.

Dr. König: This is a tremendous problem, of course.

E.v.d.S.: Of course one tries to cope with it. One certainly puts them in different groups.

Dr. König: I think there is only one possibility, and this is a kind of village school, to establish them first of all in one class until they gradually grow together. I wouldn't think of anything else, but perhaps the teachers would be about to say something.

E.v.d.S.: In my class I took the whole group together and did no second lessons with them, but gave them Greek mythology. I also painted with them, and such things, because these were things which they all lacked. And then one could begin to find one's way and to also settle them down without straightaway making demands—because those (even just those) who had had school before, were partly dead-scared of schooling and were in opposition to learning. But they were not in opposition to having some Greek myths told, and to do some painting—they loved that. And by Michaelmas they found it not so bad any more to do also some work. And then one had to differentiate.

Dr. König: You see, I have to think how different it is to repeat, say, with those who have not learned the field of arithmetic and any other subject. I wouldn't find it difficult at all to take on arithmetic at any age—at any age—if they haven't learned: because it's present. And I think it only holds good to a certain extent what you say—that it belongs to a certain age. The awaking for this special part of arithmetic belongs to a certain age. But if you are above—if you have children who are above this age—all this has wakened up, but it hasn't come to consciousness. And so you can deal with it in bulk. Can you understand? I do not say this for reading and writing, please. Because mathematics is present: it is there. It's here.

U.S.: You know, it will be different, I believe, in the coming years, for somebody to take on a ninth class in mathematics because now we all try to work very hard in the lower classes, to have really managed all that belongs to the eight classes. But now, for instance, to have a ninth class, one is really at a loss to know: 'What shall I do in arithmetic? Shall I really do something that belongs to a ninth class?'

Dr. König: You should do both, of course. Both. And you need not say: 'I have six weeks for mathematics.' Take eight weeks. And bring those who are not there in common to a level where you can deal at least with certain things of the ninth year.

F.B.: I believe we must also find a different attitude and standing to this whole question of the curriculum. We must turn to those children in the class who are the windows—who can take the lead, in order to raise the class on to the level of the curriculum.

H.v.W.: I had a child in my class who showed such peculiar characteristics. He was a boy who couldn't speak. He had fixed ideas, and he had also not a real feeling for persons around him. On the other hand, one couldn't say he was very well established in his lower senses. But it appeared very soon (when he was six or seven) that at the slightest hint of something arithmetical, he had already understood it before it was told. He was a very problematic child, and we decided then to keep back this development in his arithmetical abilities, because I had a vague feeling that it would prevent him from learning to speak. Now we worked for a time on this speaking and indeed made some progress there. But now it happened that the authorities heard of his progress and he was taken from the school and brought to an institution. There they also discovered this arithmetical ability and they made a kind of wonder child out of him. Three years later I met this boy again, and it was incredible to find that he could hardly pronounce even the numbers any more. One had to know him very well to understand that he was saying, 'a hundred and twenty-eight'—because his speaking had deteriorated very much. And I could feel—although I had worked very long with this fellow—that he had really found again his old connection to arithmetic. So I wondered what part arithmetic plays, because one had the feeling that this is a kind of balance. Somewhere in this middle is this so-called earthly ability to do arithmetic.

Dr. König: You see it is so that the boy could speak. It would be quite wrong to say that he couldn't speak. This boy didn't want to speak, out of obsessions that he had. And the repression—you were quite right in your feeling—the repression, by the obsession, of his ability to speak set other powers free, which made him able to do mathematics. Because the sound formation in speaking is, of course, also done by the astrality. Now if you repress this sound formation, another factor—which is the mathematical sound—has free play. And this he had to repress. Can you understand this?

H.H.: There is still something, I think, in Udo's first question, which one always has to struggle with—from Sixth Class onwards: that children have at a certain age missed to achieve something just in the realm of arithmetic, which naturally would have been their due to achieve. And then although one can realize it is there, they have the greatest difficulty to still develop then the ability to learn the multiplication tables.

Dr. König: I didn't deny Udo's question. Only when he asked, 'What are we going to do with this?' I'm sure there are certain times—say, between the seventh and ninth—when the tables and the repetition of tables is just the right thing. But if you miss this, you can repeat it. You can take it up at a later date. It will be more difficult, of course, because the pliability—the interplay between astrality and ether which you perform here—is not any more as easy as it is at that time. Because you must imagine the etheric body is just born—free. And now the wind of the astral body goes over the waves of the ether as you can see it on a lake: two-times-two, two-times-three, two-times-four, two-times-five, two-times-six—all that. And then you have it. Later on it's the wind engaged with many things other than the surface of the lake. It begins to stir from within—and not here.

E.v.d.S.: If one draws the awareness of the older children to these rhythms at a later age, they become aware of what tables are, and it is then a help in learning. And you wouldn't do it with a second-class child like that.

F.B.: I find it always still nothing short of a miracle to see that certain children who are dumb or unable to speak (partly even deaf children) can learn a certain amount of arithmetic—mainly, of course, with addition and subtraction—by recognizing that symbols belong to certain amounts. But once they have achieved this—that five objects get the number '5'—when they then come to doing additions with up to three digits, there is something of a sound-experience.

Dr. König: Can you describe it a little bit more?

F.B.: There is Susan Bean, now in Camphill but no longer in school, who is deaf and mute, also psychotic. There is also Mark Hawley, who hears but cannot speak. Sandra has learned to associate numerals with certain numbers of objects, and gradually learned with numbers up to 9 and then up to 20—and then very soon up to 1000, with the help of abacus and whatever. And all that she needed was encouragement, because she is very fixed in what she does and can come forthwith, but she did additions and subtractions quite correctly, and quite obviously was tremendously interested.

Dr. König: What did you say was the sound-experience? I couldn't understand this.

F.B.: You know with such children where there is no manifestation of sound—also Mark is a boy who cannot at all utter a sound—one gets the impression in doing (in manipulating) these numbers, these processes, there is a sound-experience in their number-work.

Dr. König: And this is the sound of which I spoke: This is the sound of the astrality, which is always there—and this you somewhere experience.

F.H.: When you speak in your article on the sense of movement (on the *Bewegungsvorstellungen*), patterns of movement which we imagine before we execute them, are these the same as the sound-patterns?

Dr. König: Yes. The melody of the sound-pattern. The melody of the sound-pattern.

I.R.: I would like to ask still something in connection with the mongol child. I found that the mongol child really has such great difficulties in arithmetic because it would so very much belong one-sidedly to the part of harmony—to the part of the thing which does it: because of the mercurial element in these children. And I wondered, therefore, whether in various types of children one could see that the one side is so very dominant with them. I could, for instance, imagine that for a more withdrawn child—yesterday we spoke about the passive and active part in a sum—that they would much more belong to the passive number, to the passive part of a sum; and therefore because they are so much in there, they can't reach the more active part of the sum. Is it a possibility that one would try to recognize in the children whether they are tending towards one element too strongly, and whether one can then in some way help them?

Dr. König: I think this is a little bit off the rail. Because in the mongol child it is obvious that the lack of ability to do mathematics is connected with the very late motor-development—and the very late motor-development is due to an all-round flaccidity not only of the muscles but of the whole of the sense of movement. There you can see it as obviously as anything, that the sense of movement was not formed out by the astral body—it didn't come to grips with the sense of movement. It came to grips much too late with the sense of balance. And therefore the enhancement of the lower senses doesn't come about. This is the main thing. They have a very vivid sense of life, and so they learn to add and subtract sometimes. But anything which goes even half a step further is impossible, simply for the reason that the lower senses—the sense of movement and also the sense of balance—are slowly aware. And you can see how coarse in their movements they are. At least I see it so.

I.R.: So you would not think that it is a real thing to see children connected especially to one of these three parts?

Dr. König: Oh, yes—this I wouldn't deny. Only what you said about the mongol I think is incorrect. That various children are connected with different parts I'm quite certain. And it will depend on your fantasy—on your possibility to be mobile in your fantasy and thoughts. I hope you can do this in the class better than here.

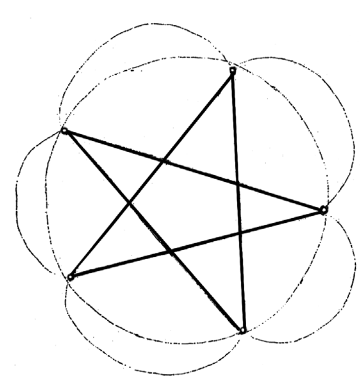
H.C.V.: I would like to come back to what Ingrid said, because I think it gives an angle to what Henning meant—if I understood him rightly. I think it is so if you introduce the tables—the learning of the tables—at a later age, it is difficult to do it still in the same way as with the seven-year-old. You can't do it any more in the purely rhythmical ordinary way, but you must try to fix the sound patterns on the blackboard with colors—so that it becomes visible.

Dr. König: Very interesting.

B.H.: My Sixth-Seventh Class was rather bright but unable to do tables. In order to find a way I had the idea of using geometrical patterns on the blackboard. From there we went on to sewing the tables on wood with laces. And first of all—especially for the very backward part of the class—it was just an activity which needed the skill of the hands, which could be difficult enough. But as they went on, they would suddenly realize what they were doing, and would come and say: 'You know what I'm doing? I'm doing the two-times tables.'

Dr. König: How does one do the two-times tables? What form is it?

B.H.: You see, as we were better in geometry than we were in arithmetic, we went out from the circle. And there we made ten points. And now we had in out copybooks already—by the help of young Johnnie, who used the table—learned that when we had the two-times table, we had to skip one and stop on the second. So we did it: We skipped the one and came here—and again; and so we went on. And, of course, we got a geometrical figure.



Dr. König: Lovely.

B.H.: And then we could not only do it like that, but we could, of course, also take such a skip—like this. And then from there we made wooden boards with the circles and the holes—and the children were finding their way. And not only the two, but in the three, the four and the five—and, of course, they got different patterns.

Dr. König: It's the same, exactly the same. Only in a different way. What have you done? What did you make the children do? Instead of clapping, what did they do?

E.v.d.S.: Touch?

Dr. König: They clapped with their eyes. Instead of here, you did it here.

B.H.: But through that the first dawn of thinking came in some of them.

Dr. König: No doubt.

B.H.: And we could then—I mean, now the first two groups—do arithmetic: mixing the Four Rules, and with problems not put in figures but a problem put in words which they had to transfer into figures.

Dr. König: Excellent. Excellent. But you see, you others shouldn't repeat this—but find your own methods. Not because this is wrong, but there it works. If you repeat it, you are just a repetitioner. You must work out of your own fantasy, together with the class.

J.S.: Do you think then that one should appeal more to the audible at an earlier age and the visible later on—or something like that?

Dr. König: Not 'one should'; one has to. You have to apply not the audible thing but the rhythm in the lower age; the eye further on, and in the end, the hearing.

J.S.: So it's the touch more than the rhythm?

Dr. König: The touch, the sight, the hearing.

J.S.: But with the rhythm you have the more musical quality—not really connected with thinking.

Dr. König: It always has a musical quality—also with the eye, because you do your clapping with the eye. Only at the same time it connects itself with this. This goes from seven to about sixteen or seventeen.

K.v.S.: In the Sheiling I visited the weavery. Thesi described to me now she had experienced that children who in school had not been able to do number-work, now in this different situation where they had to count made quite good progress in arithmetic.

Dr. König: But if the teacher had done this already when the child was 8 or 9, Thesi wouldn't have needed to do it when the children were 16 or 17. Dear friends, don't sit too long on your chairs and muse. When you do mathematics, you must move: jump, dance—all that. Without it, it does not work. With normal children, it might work, but not even there. If you think, then you can teach mathematics—certainly not; it's quite impossible.

J.S.: But we must do it, not the children.

Dr. König: You must do it with the children. If you are able to do it inwardly as vividly as the children do it outwardly, then it's all right. … You shouldn't mix up the movement—but in order to make them do mathematics, you must prepare them always. And before this, in preparing them, you should use movement. So that something begins to stir, shake the bottle before use. Shake your own bottle.

K.v.S.: I have done through a number of weeks now in my class every morning ten minutes of mental arithmetic, which was built up in such a way that I knocked the number, they had to listen and show with their fingers. I can now almost do the Four Rules mentally. I let them do it very quickly with their fingers first, the number I knock—also subtraction and whatever—with this knocking and with the fingers. And very quick reaction.

Dr. König: Excellent. It is always necessary to connect two or three sensory experiences, be it sight and sound, or sight and touch, or touch and sound. This inserts it. This makes it real. If you do this [showing four fingers], but if you do this [four knocks] at the same time.

F.B.: Has that to do also with the force of antipathy? That by my calling on different sense-impressions, one calls on this?

Dr. König: Most certainly, and therefore you raise the consciousness.

F.B.: I didn't understand the beginning. We always feel that children who have not got the forces of antipathy have difficulty in arithmetic. In calling on different senses to partake, we invite the forces of antipathy and then bring about consciousness?

Dr. König: Yes.

U.S.: I have the impression that we as teachers have to find some sort of Slotemaker scales for arithmetic. Not now tables, and down it goes and nobody has much of it—but much more in applying now to all four rules (as you said, Karin) and have it perhaps daily, and if for five minutes, to keep something alive and keep something at work within a child, which might then be available if we then have again arithmetic.

Dr. König: Why not? But you shouldn't say 'we as teachers.' You should say, 'I do it.'

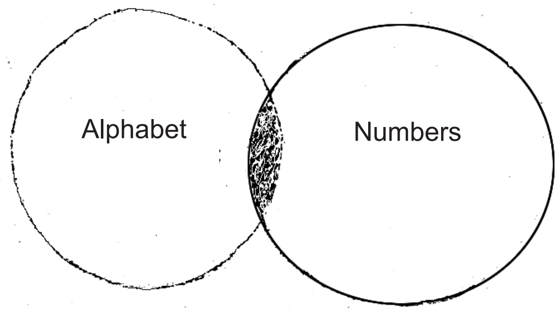
K.v.S.: There's something which has puzzled me for years now. I can't get to grips with it. There's one child in my class who can count. She has also learned the letter forms (that means she can recognize forms), but she is totally unable to connect the number forms with the number—and I am at a loss. We have tried everything.

Dr. König: Could you describe it again—because it doesn't make sense to me.

K.v.S.: It is Sheila Richardson. She is able to recognize letter forms. That means she can recognize a complicated form and the name of it. She can also count. She can also possibly count and bring you five or six objects. She will count it: 'One, two, three, four, five' and bring it. But she is unable to recognize the symbol '5' and connect that with five objects.

Dr. König: Well, now I understand. This is quite well-known: that children, for instance, have the ability to recognize words, letters, but as soon as they read or should read a number, they are blind to it. It's called "number-blindness."

K.v.S.: But I find it difficult to understand because she can recognize forms—and that, in my eyes, is a form.



Dr. König: But have you not so far understood and found out that a letter is something entirely different from a number? As a symbol, entirely different. You have a part of your mind which recognizes, let me say, the alphabet (to say it quite bluntly). And it is an entirely different part which only somewhere overlaps for the recognition of numbers. Completely different. And when you read numbers, you read numbers entirely differently from the way you read letters. Don't you know that you read letters like this [left to right] and numbers like that [right to left]? In German we say 'vier und siebzigs.' We don't say 'seventy-four.' And this is because we read now from right to left, and we read letters from left to right. And if you now investigate this child, or have investigated it, you will find there is a mathematical defect. And a very special one. I can't say which one. Probably from eye to hand. And therefore the reading of numbers is without any recognition. The world of numbers is something entirely different from the world of any kind of words, vowels, consonants—something entirely different. You build up with 'one, two, three, four, five' something completely opposite from what you build up, say, with 'A, B, C' and so on. You can't say: 'This is a symbol and that is a symbol.' Then I would have to say I wouldn't be able to shave with a knife with which I have to cut meat. Then you would come and say: 'Knife is knife.' It's not the same.

K.v.S.: But how do we recognize then such a symbol?

Dr. König: There you would have to discover certain things which do not belong in this Conference. You can ask and we could once speak about the recognition of symbols, but this is an entirely different thing. There we would deal with an enormously complicated subject concerning the sense of word, the sense of thought and the sense of ego. This is something entirely different. I have nothing against it. I shall be pleased to help you in whatever way I can. Only let's do first things first.

H.C.V.: I would like to say that this task which we have put before us has not gone past—was not forgotten. But it is a very formidable task. Perhaps one could mention, I feel, that we must take hold of these operations first of all as processes—I think quite apart from the fact that they have a beginning and an end—and see how we can build this up in the children: that they understand the process—something in time.

Dr. König: I am convinced of this. I haven't spoken of time, because this would be another step and would still be much more complicated. But I could imagine that if some of you would now start to make your experiences and carry this into teaching and come back again in a year's time, we would then be one step further. Because I'm sure you are right, Hans Christof, that one would have to regard especially the processuality: I would say, the rising and falling, the major and the minor. And when you start with a sum and you are adding, you are in the major scale; when you start with a sum and you are subtracting, you are in the minor scale. But you wouldn't manage with multiplication and dividing because there you come already to tact. You can do without tact (I mean beat)—you can do without beat in adding and subtracting, but you can't do any more without beat—the actual beat, the rhythm (threefold, fourfold, and so on)—when you multiply and divide. At least this I think. Perhaps it is not so.

H.C.V.: I'm not so sure. I mean, it belongs to the characteristics of whole numbers that we cannot get hold of them unless we take the rhythm. I mean we can never get hold of a number as a whole: We must always rhythmicize it. And we can take any number system—as you pointed out, not only based on the ten but also on seven or on twelve; but we must rhythmicize the whole.

Dr. König: But rhythm is not beat. [Beat is tact.] Rhythm is necessary, I fully agree. But whether beat is necessary—this is the question to me. I wouldn't say it for definite. I would certainly put a question mark behind—whether or not. But it would be on you, on many of you, to find out. This would be so important. Because I am sure we have come with this to a fundamental discovery, which perhaps was already made and I don't know it. But it is something fundamental—especially for the teaching.

H.v.W.: Just because of what was put in front of us this afternoon, I felt very sharply the contrast between how arithmetic can come alive as an experience and what part it plays in our time. Because one can say in general that arithmetic plays a deadening part in life, if we think of statistics and the general trend of putting everything into quantities. But how far are we obliged to make alive for the children the quality of the number?

Dr. König: Well, you see—this is, of course, an entirely different chapter from what we were dealing with. I'm convinced that it is a most important chapter—to come to an understanding of the quality of number—because this is the only reality in a number, and I am under the impression that many of you in classes begin to teach arithmetic in trying to tell something about the one-ness and how it comes into two and three and so on, which is a certain foundation of what you do in your lessons. Or is it not so? I would like to hear your views.

N.J.: Yes, I feel there we are moving towards an answer to questions which showed themselves at the beginning—in the discussion we had before you came: because we learned of this headmaster of an E.S.N. school near Bristol who has been talking to the friends at Thornbury and has impressed them with his sincerity—where one could feel that here was someone (like many modern teachers) who knows somewhere within himself that 'teaching must bring the child to reality—the child must deal with what is real to his experience.' And yet then Thomas had to say that what this headmaster was doing, if it would carry on, in three generations would turn the children into animals. Because actually the reality then that such teachers see is an entirely materialistic reality: lengths of wood and such things are a purely quantitative approach to mathematics.

Dr. König: Applied arithmetic.

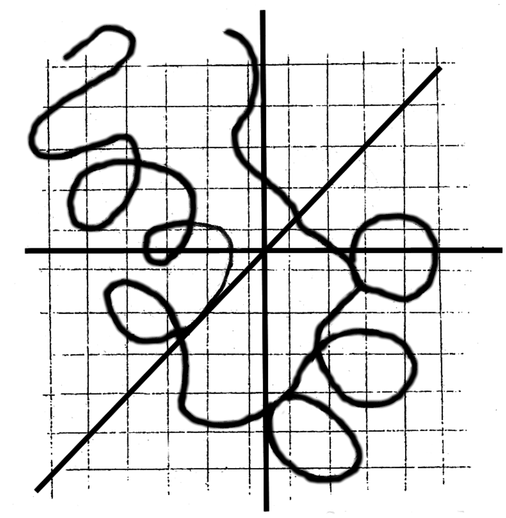
N.J.: And I feel that what you have said has helped us all to have a renewed conviction of the spiritual realities of arithmetic and of numbers themselves—and that this is the way we must begin actually. I find that I feel into geometry very early for this reason, because in geometric patterns one experiences the wonderful mysteries of 3, of 6, of 12.

Dr. König: Of course, yes.

K.v.S.: Just as an answer to your question about what we do about the qualitative number, my experience was that we mainly carry on number-work in the second lesson and that after a year or two, it comes to a dead end. And numbers are, so to speak, without use, I mean, they are not any more really alive. And at that point I put a period on the qualitative number again in the main lesson—to re-enliven the second-lesson work. I would agree with Nicolas that then the geometrical approach is a great help to come into the qualitative appreciation of the number. There should be every now and then a main lesson which is continued in the drill work in the second lesson.

Dr. König: Would it not be very useful for you, for instance, after a year to meet again and to turn to the quality of numbers? I think it would be something very wonderful. So that not old patterns alone, but new ones would be achieved. And I think you should invite one of the great Waldorf teachers to come and tell you something. Bindel, for instance—he would jump at it, I'm quite certain. I feel you should actually as a Teachers' College meet every year. It would be so important. And invite different people. Not that I wouldn't like to come, but I have the impression Bindel could tell you immensely more than I can in this subject.

U.S.: To come back to what you said before, 'If you see this, you will see why children should weave, spin and knit.' Now I could think that it is always very necessary to let children knit and weave—I wouldn't say in main lesson, but so that one also thinks now they will have a different approach to arithmetic, to what is the realm of difference, I thought. Could one also find other stimuli for what is proportion and logarithm? Could you give us any advice there?



Dr. König: No, I couldn't.

U.S.: I had to think, for instance, of what Elisabeth does: that she does a lot of musical exercises. And I thought that might also be a great help for arithmetic. Beating and so on.

F.B.: I think it is very important what came up this morning in the discussion: Whatever aids we may find, they will have to be found by each of us in a different way.

Dr. König: I think so.

F.B.: We may not find geometrical patterns such as Birthe's. I also know that dynamic drawing, which Kirchner does, is a direct help for his children's arithmetic. We can also do these exercises with our children, also knitting, and it may help arithmetic. But we should do them at their own time.

H.v.W.: One should not copy the dynamic drawings of Kirchner, but re-create them. I think they could work also in the hands of others if they were re-created, if one would know the 'why' behind the forms.

Dr. König: Udo, you still are under the impression (like millions of people nowadays) that the remedy works out of its own power. But this isn't true. If I give cardiodoron and another gives cardiodoron, and a third one gives cardiodoron out of the same bottle to the same patient, it will work quite differently.

U.S.: That is new to me.

Dr. König: Therefore I say it, Udo.

U.S.: Only I did not want a remedy. Only I wanted some directions.

Dr. König: I know you don't want a remedy. I only tell you, you won't find remedies which are good forever and good for everything. This doesn't exit. Rudolf Steiner once was very, very upset—for the following reason. In the Society (it was still before World War I) remedies were known to be made by a lady called Ritter—Frau Ritter. She was tiny, thin and anything but beautiful. She was able, for instance, to hover over the ground (a few inches above it) without walking—just went up. Now she made the most wonderful remedies—ointments which we never have had since. But the demand became so great that Rudolf Steiner said, 'Very soon these remedies won't work any more because too many of them are used and their power will go.' We experience this continually—even with penicillin: it has worked for a few years—now it hardly works any more. And so if I tell you: 'Do this, in order to enable in the child the logarithm,' I would be a very, very bad doctor. Can you understand this?

U.S.: It is hard, but I try to.

A.P.: You know what I have found is that I see what other people do. And if I hear what they do and I see it, out of this I can find my own remedies—which I couldn't have found if I hadn't see everybody else.

Dr. König: That's the thing. Most certainly, you should make as many experiences as possible. But, you see, 15 years ago, when you came into a conference of anthroposophical doctors and you entered from behind, you could see only their backs. They were writing down prescriptions which the speaker told them. My intention and Dr. Schickler's intention was to raise their heads; and during the last 5 or 6 years, not a single prescription has been written down during a medical conference. Can you understand? Because it's useless: If you don't have the great images as a doctor and then use a remedy out of the situation, out of the spur of the moment, out of the presence of mind. You can't say (like the Weleda) 'Aurum is good for the heart.' It's nonsense—complete nonsense. Because you can also say Stannum is good for the heart and Silver is good for the heart. Everything is good for the heart—if it is given in the right spirit, in the right form, and for the right patient.

N.J.: And this is what education must be.

Dr. König: Of course.

N.J.: You know, if one goes to a Waldorf school now, one sees the same main lesson being given that was given 20 years ago.

Dr. König: Exactly.

N.J.: One sees the cuttlefish on the board, and such things—

Dr. König: And the mouse!

N.J.: And you know one feels that there is a touch of death in it. And the children don't take it up and react simply because of this: It's the same old medicine. Now there's no doubt that this is connected with realities which are eternal realities. But I think it is (I mean, it is a constant question to me) how always to re-create these things when one has been given this sort of Ur-image. But there are many Ur-images actually.

Dr. König: Of course. I mean the whole animal kingdom is an Ur-image, and you don't need to rely continually on the cuttlefish. I've said this for years, and I'm glad you say it now.

K.v.S.: I agree fully with that. But when I had this period, I had decided not to take the cuttlefish just for this reason.

Dr. König: And then you did it?

K.v.S.: And then I looked around everywhere—and in the end I came to the conclusion that the cuttlefish was a very good example!

Dr. König: But you see in the moment you do this, you have re-created the dear old cuttlefish. It is an incredibly lovely creature to describe.

N.J.: The important think is not to do it because it's the done thing.

Dr. König: That's the thing.

N.J.: But I was horrified to see a teacher with the whole main lesson written out, you know—and obviously it's handed over from teacher to teacher, pictures and everything worked out.

Dr. König: It's exactly the same as our doctors going about with the Heilmittel-verzeichnis in their pockets. It's issued by the Weleda. It's a book of 350 pages; you can carry it in your pocket and look up: 'Sore throat,' 'Pneumonia.' It's quite easy—and it even helps sometimes. As soon as you are enthusiastic in writing out such a remedy—perfectly all right.

H.C.V.: To the method of introducing the Four Rules, I would like to mention once more what I mentioned also last night. Dr. Steiner very much wanted the teacher to have in the background of this method an understanding how we as human beings altogether think. And what happens there that we unconsciously—out of this present world of number—break down and make it into a concept, and only then can arrive consciously at an understanding that we put that in front of the child in having the heaps of ones and putting it apart. I think that is exactly the same.

I.Ra.: It's perhaps a little off the trend, but I feel that I should come back once more to that what Dr. König said when he talked about the three means to teach arithmetic: to take the help of the touch (of the sense of touch), the help of the sense of sight, and the help of hearing. This is so with so-called normal children. But I have very many years taught deaf children, and now recently watched a great deal how the blind child was taught—partly also took lessons. And there I was now very much struck by comparing how what has worked in these two types of children. When I think back to the deaf—with the limited deaf children whom I had to teach—how they immediately in arithmetic come to combine their touch experiences with their sight experiences, and that is a very quick process. Take a child who has come to school at seven years, and who has managed to talk and to converse properly by the ninth year. In him this process is concluded very quickly. You can't do any more except widen the scope a little. I had to ponder because we have in the sense of touch, so to speak, the representative of our bodily experience, and we have in the sense of sight one sense which is very much connected with our middle senses, with our soul. For the deaf child, there is no sense available which would represent the spiritual part of mathematics, the hearing is not present, it cannot be used. The deaf child takes hold of this experience very quickly; he loves this subject and goes into it, and conquers it. But it was more from the utilitarian aspects of arithmetic, those aspects stressed by the headmasters of the E.S.N. school who visited Thornbury: the use and the handling of numbers. In this realm it is conquered and they know the quantity—but they have no idea of the quality of numbers. I have tried for years to combine Religion Lesson with bringing the qualities of numbers, and I have completely failed.

Now having watched Elisabeth for ten years faithfully teaching our limited blind children, I see the opposite happen. Their sense of touch can give them many other possibilities. For instance a very limited retarded blind child would be able in Austria to touch a trolley and in a quick movement going over the trolley would say, 'Look, the trolley from Heathcot has come here.' And it is really the exact trolley which is made the same shape, the same wood, the same polish, which the child experienced seven years ago (let me say) in Heathcot. But the sense of touch cannot be used for finding out: 'Aha—here are five bricks.' It mostly does not take in three or four, because what is not held together that is again one, one and one. And we have studied this ability and inability. On the other hand from the sense of hearing, mathematics comes down and never quite unites with the tangibles. Only when the child gets into puberty the soul-life is consolidated and contact is established through the walls of the surrounding. One can now feel that what Dr. König has called 'standing between light and gravity' is now at last established seven years later. He would perhaps get a little glimpse of this mathematics which he had already achieved years ago as a quality in knowing, for instance, 'That's a second,' 'That's a third,' and 'This is a fifth' in music. Mathematics had been for him merely a qualitative experience.

Dr. König: It's very interesting, very, very revealing.

I.Ra.: The deaf child—and we have many children who are not just deaf physically—must be protected from finding the answers too quickly and must be shown the many other possibilities. Always one would like to push away the conclusions. Whereas for the blind child—in order to get at least a few glimpses of light—we have to pull down continuously that which is a sound and quality experience. One can feel the blind child embedded in this, as in a huge cloud.

M.M.: Doesn't the blind child—in what Ilse refers to—use the sense of touch as an experience of memory and therefore he cannot get hold of mathematics with this sense? Does not the blind child memorize in his touch? And does not the deaf child memorize with his sight?

Dr. König: I don't think so.

F.B.: I am still puzzled by some children who are in harmonious possession of their senses. There are some especially of the more psychotic type, who have a certain flair in grasping number combinations, who can mentally calculate very great numbers—who even often have an ability which we lack, in seeing a great amount of objects at a glance to say, 'There are twenty-nine.' But they are maybe the ones who are very much in the light, where the higher senses are more engaged than the lower senses, where the enhancement is a very sudden one.

Dr. König: I don't think these are the higher senses. I don't think so. Negroes, for instance, who have no possibility of any kind of number-work, but when they are herdsmen are able to at once know whether all their cows (up to a number of 2000) are together or not. If one is missing, they would at once know it. Up to 2000! I say this because experiments have been made.

I.Ra.: And that is not supersensible?

Dr. König: Something is still alive in them. The senses of touch and sight and hearing are so wide that they have it, so to speak. You see, Rudolf Steiner once said in a lecture (I can't remember when it was, but I remember the words very well because it was so impressive to me) that the amount of darkness in the world shows exactly how much we have done in arithmetic and mathematics. The amount of darkness in the world is parallel to the amount of mathematics—which one can understand—because the primitive simple light of everything in our senses is darkened by putting it into numbers. And as soon as this is done, this old ability is gone.

I.Rö.: In the animal world it is probably also experienced so? For instance, a flock of wild geese would know if one of their number fell out?

Dr. König: I'm sure; I'm quite certain. Exactly.

I.Rö.: Because they probably also have this wide consciousness.

Dr. König: Yes, they are conscious of that. And this is not supersensible consciousness. Their senses are still so unharmed—so wide—that there it is. Like dogs, for instance, smell things which we are unable to smell.

H.v.W.: Dr. Steiner gives also an example of certain tribes in Africa with a sense of smell. Without seeing somebody, they smell if it's an enemy or a friend.

Dr. König: Of course, of course.

F.B.: Is not this tribal ability to be compared to the child psychotic? Is it not that their light is still so great? But on the other hand, we have numbers of children who are 'feeble-minded,' who are more in the gravity, in whom the darkness is greater than the light. They are bound to the pre-number activity that we do. For them mathematics is not yet awakened. Then there are the others who have it 'at their fingertips.'

Dr. König: Which is the blind and the deaf child, only in metamorphosis. And if we would again keep this as a kind of guidance, it would be very helpful.

H.C.V.: Only as a picture on the other side of the scale: An engineer who is asked to say suddenly what is 2 times 2 takes out his slide-rule and finds '3.95, roughly 4.' Our thinking has become so lazy.

Dr. König: Of course: they do this really. You know, there is one think I would like to add to this which you have just said. If you have the child—let us say the dark child, we understand each other now—the deaf, dark child. I am convinced you would have to do additions starting from the '1' and going to the powers. But if you have the autistic child, the light child, you must start with the powers and lead down to the '1', because you must train this child in the opposite way as the other one. And whatever Rudolf Steiner says is quite right for the normal child, but we should feel free to metamorphosize it. Because when you start with the '1,' you simply touch something which he anyway knows and he is not interested. But if you come from here and go up, I am sure you would put in a remedy which would be helpful. One can try it.

F.B.: It is interesting that Rudolf Steiner said that with the phlegmatic child one should start with the '1' and with the choleric, the powers.

Dr. König: Of course, quite so. What the one does, the other has to 'undo,' so to speak. And this one should follow much more. The way in which Bindel does it is too intellectual.

F.H.: Since this morning I've had a question which comes up now again in our discussion. And that is: In the development of the child (what you describe, for instance, in this surging up of the blood), there the development goes from the walking to the talking to the thinking, and is a process upwards. And also what Rudolf Steiner describes then as the growth of the child in its inner development goes the same way. But what is now the arithmetic process—the development within the arithmetic, the learning of arithmetic—is the process which goes the opposite direction, which goes from the dead downwards, from the thinking to the feeling to the willing. And I wonder whether there is any connection between this and what Rudolf Steiner describes in the "Meditative erarbeitete Menschenkunde," between the Apollonian and the Dionysian streams, in connection with this light and darkness and the plastic formation and the music formation in their polarity.

Dr. König: And, you see, just the music streams upwards and the image streams downwards. And I would never agree if you say 'walking, speaking, thinking.' I would say: 'Yes, but also this.' And exactly the same for the mathematics—'Yes, but also this.' These two have always to come together. Only generally one can say: 'will – music; thinking – image, formative powers': from above to below. But each single processuality is always both.

F.H.: With these children who are spastic, also those who are bound more to gravity, who can't jump, who shuffle along, who are stiff in their movement, and who also have very often no possibility to learn (quite apart from numbers)—one can see how the whole musical element is completely fettered; whereas the children who are light—there again, their understanding is very quick, but then the application is quite uninteresting.

Dr. König: Quite so.

K.v.S.: Already on the first day you spoke about the teeth, and you said that first there were 20 and then 32, and then you said this is mathematics within us. And then today again you brought this image of the fingers and showed us with these numbers how that is. Now this is the phenomenon. But now again a very naïve question is, 'How is that so?'

Dr. König: Well, first of all as soon as I told you 4 + 3 is 7 and 4 x 3 is 12, it has already gone together. Because you understood it, didn't you? What you are asking is, to put it in simple terms, 'In what way did God know that He should create the five fingers?' And there I would have to say you must ask Him. I'm sure He will tell you. … But He will tell each one another story, not the same story. Because each one will be able to grasp only one side of the story; and God is so immense that He is able to make this side into a whole story.

K.v.S.: I thought just the answer, 'Der liebe Gott hatte es so gemacht' would not quite be enough.

Dr. König: But if I wouldn't have drawn your attention to it, you wouldn't have come to this question. And that's something, isn't it?

K.v.S.: Yes, indeed.

U.S. During the night one is sometimes wise than during the day. Mathematical problems can be solved during the sleep—that has been proved. How far shall one or can one make use of this fact with one's children?

Dr. König: I only say: 'Not at all'—we have no permission to do this. Because then you could also say we hypnotize our children and tell them everything, and then they will be able to know it.

I.Rö.: But it's a different kettle of fish if the child knows, for instance, the sum he will do tomorrow. That is a help.

Dr. König: Certainly, but then he recovers in the night and does it consciously. But he means, for instance, how many mathematicians found the solution of their problems in sleep. The greatest mathematicians dreamt the solution of a problem, and then when they were quick enough to remember their dream, they knew it. Didn't you know this? Most of the great mathematicians dreamt the solution of their problems.

I.Ra.: But that would be most dangerous to use.

Dr. König: Therefore I said a conscious person can do it, but for a child it would be impossible. Didn't I ever tell you this story of Agassiz, one of the greatest zoologists? I never told you this story? He found the fossil of a fish—and at this time (it was the middle of the last century), paleontology was not very well-known—and when he saw the fossil of this fish, he was unable to find the classification. Then one night he suddenly saw this fish swimming toward him, opening its mouth and telling him the scientific name and where it belonged into the system. And poor Agassiz woke up and had forgotten. But he didn't give way, and the next night he said, 'Perhaps the fish will come again.' And he put a sheet of paper and pencil under his pillow, and the fish came then, and he told him again the name. He woke up and wrote it down and looked it up—and it fitted perfectly. And so any entirely new discovery was made in the whole system of ichthyology. So many, many things are discovered. The formula of benzole, for instance—which was a revolution—was discovered by Kekule. It was at the beginning of this century, and he was a very famous man. He was riding by coach out into one of the suburbs of Copenhagen, and it was in the evening and he dropped off to sleep. Suddenly he saw six tiny Negro boys holding each other's hands and dancing around. And he woke up and knew: This is the formula for benzole. This is the way most of the great discoveries. They are not made in waking life; they are made in sleep, in dreams.

I.Ra.: They actually do suddenly and in one moment what altogether has to happen in mathematics: that it is raised out of our darkness.

Dr. König: Of course—exactly. Kekule would never have dreamt this without having put a tremendous effort into finding this formula. Only he wasn't clear enough, so he dropped off and out it came, and there it was. It's unbelievable!

K.v.S.: I believe to Udo's question one could still say that Rudolf Steiner brings these examples of how the night experience can clarify the impressions of the day. And he mainly brings it up in connection with geometry. It is a great responsibility to know that what one does during the day does indeed have an effect during the night.

Dr. König: Of course—but Udo means something else. He means the transmitter here, at the back of the head, saying 'two times two, two times three, two times four, two times five'—and so on.

U.S.: I meant that if the children themselves have not enough light during the day, whether it can become light during the night. Is that possible?

Dr. König: It must become light during the night—but you have not to influence this. Let it become light.

U.S.: And the tremendous effort is all right during the day? That's specially what I thought.

Dr. König: Of course. This is most necessary.

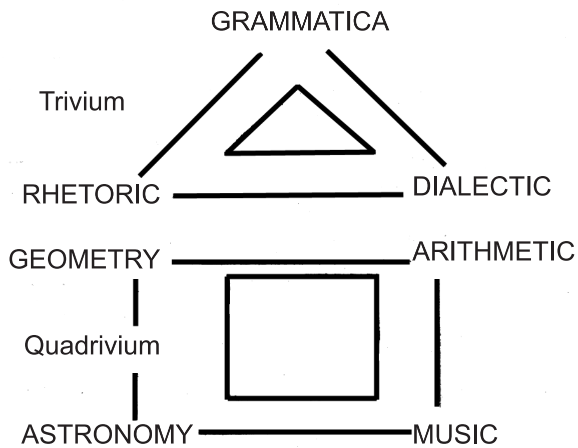
Conference 2 – Seminar I

Dear friends, as Friedwart has pointed out, it is now two years since we were together to speak on Arithmetic. There should have been a continuation last year, but this was not possible because of many other things I had to do—and I also thought it might be good if you would start with some of the many tasks which my lectures have produced for you. Now Friedwart has pointed out that in spite of this you would like to be burdened by some further tasks—and I will try to oblige, though I warn you, from the beginning on it will be very stiff and hard going.

You have asked me to speak about the quality of numbers. I am unable to do this as a mathematician, because I am far from understanding even the fundamentals of arithmetic and mathematics. But I am willing to speak, as a doctor and as a natural scientist, of the revelation of numbers in the works of creation—and thereby help to find a way to the sphere whence numbers emanate, to the sphere where numbers in their qualities come from, and when they permeate the seen and unseen, heard and unheard universe. So that we might in one way or another begin to divine something of the real quality of a 6, of a 5, of a 9, of an 8, of a 2. You understand I don't say "of 6, 5, 9, 8, 2", but "of a 6, a 5…"—because this is a being. Not that we will be able to face this being, but we might be able to get a first indication whether this or that number is (now I speak in pictures) a tailor or a shoemaker, a joiner or a smith, or even an artist. In this way we shall try to understand each other. And I think it is a good choice of subject after what we occupied ourselves with last time.

You will remember that two years ago we spoke about arithmetic procedures—adding, subtracting, multiplying, dividing and so on—and tried to understand not the actual doing but the foundation which addition, subtraction, multiplication has within the human being. We need only remind ourselves of the quality of the lower senses in connection with the possibilities of doing arithmetic. We started to divide at this time; we went into addition and subtraction, multiplication, and so on. All this we tried to understand.

Our task now is to understand not the processes themselves, but the qualities wherewith these processes deal. We need numbers to add, subtract, multiply and so on, but now we will look at the numbers themselves. The processes of counting, reckoning and so on have their primeval source—have their wellspring—in the numbers. You will not so quickly understand what I mean. Here, as it were, we deal with the processes of arithmetic. Above there are qualities of numbers—and out of these qualities have emanated adding, subtracting, multiplying, dividing everything else. How is this to be understood? The numbers have created the processes of arithmetic, in order to be dealt with by these processes—in the same way as an unborn child leads the parents together, in order to incarnate itself through them. It is difficult to understand, dear friends. But we are not together of three lectures. We are here to discuss certain things which are at least as interesting for me as they are for you. If you hadn't asked me to speak on numbers, I would never have thought of occupying myself for a whole year with this subject.

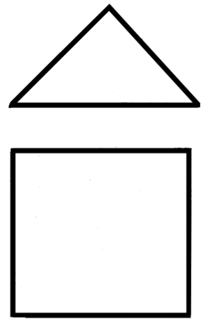


I will go one step further. Round about the 4th, 5th and 6th century after Christ—but then continuing right into the Middle Ages—students in monasteries and later at the universities were studying what were called the Seven Liberal Arts (die sieben freien Künste). The study of the Liberal Arts was meant to make the one who studied them a free man. A free man. He did not learn a craft, but he learned the Seven Liberal Arts—in order to conduct within his mind a process similar to that which every human being goes through during his first three years, when he learns to stand upright, to speak and to think. It was an enhancement of the achievements of the first three years of childhood: It was now the human spirit that should stand upright, speak and think. In these Seven Liberal Arts the flower of what men achieved in the course of the Fourth Post-Atlantean period was handed over to the beginning of the new epoch—the Fifth Post-Atlantean Epoch, the epoch of the consciousness soul. Beautifully, all the spiritual achievements of the Fourth Period were brought together into one seven-petalled flower—the Liberal Arts. The word was coined in the 5th century, by Martianus Capella. He was the teacher of the individuality who was later born as Campanella, and still later as Otto Weininger. Round about the year 450 (it isn't exactly known), he wrote a poem in Latin called "The Wedding of Mercury with Philology"—in which the Seven Liberal Arts appear as beings. This was taken up by one of the greatest philosophers of that time, Boethius, who divided the Seven Liberal Arts into three and four: the Trivium and the Quadrivium. On top of the triangle was Grammatica—as grammar it has come very low in our time, hasn't it? Here was Rhetoric, and opposite was Dialectic.

These three Liberal Arts—the Trivium—dealt entirely with the Logos, the Word: and they are still alive, though for centuries they haven't been regarded as anything real. The Quadrivium is different: It consists of Geometry, Arithmetic, Astronomy and Music. Astronomy is not what is today considered to be astronomical, nor is Music pop or jazz or record-playing, nor is Arithmetic counting, or Geometry drawing. Quite otherwise. In Platonic bodies we really meet geometry; the light which builds space mirrors itself in such fundamental forms as the cube. And all this is also number. On one point in a cube three squares meet—and this is their only possibility of meeting in three-dimensional space. In a tetrahedron three triangles meet, but in an octahedron we have a figure in which four triangles meet, and in an icosahedron five triangles meet.

If you begin to listen into such forms, you will have a first inclination towards what Geometry, as one of the Quadrivium, once meant. It is similar with Arithmetic: Arithmetic was the creative power of the quality of numbers, which reveals itself in the different forms and processes of reckoning. If you open the veil of Arithmetic, you are able to hear it—and this is Music: Rhythms and melodies are nothing but mathematical operations. If you open the veil of Geometry, if—as Kepler still could—you would open the veil of an octahedron and look behind it, you would perceive the music of spheres, according to which the stars are going their courses. As the Trivium deals with the word, the Logos, so the Quadrivium deals with the quality of number. By learning to identify oneself, as far as one humanly can, with both the word and the number, one attains a certain amount of freedom. Freedom not to do what one likes and avoid what one dislikes, but the freedom of being an independent spiritual being. And the teachers in the monasteries—for instance in the great School of Chartres—know: "If I can bring Logos into connection with the number, I can attain what Aristotle with his ten categories tried to make into Logic." Somewhat like this one could describe it. The Liberal Arts were not a fancy business for the few, compared with us very low-minded philosophers. They were the reality of what was carried over from the 4th into the 5th Post-Atlantean Epoch. You see how important it is to get an idea of what 'quality of numbers' really means! … The Three is the door into the mysteries of the Word; the Four is the door into the mysteries of Number.

Today Geometry, Arithmetic, Astronomy and Music have metamorphosed. We would no longer be able to enter through the Quadrivium into the mystery of number. Geometry, Arithmetic, Astronomy and Music have put on an entirely new garment through Natural Science and the development of the 5th Post-Atlantean Epoch. They are now wearing the garment of phenomena: geometrical, arithmetical, astronomical and musical phenomena. At the time of Boethius, at the time of Martianus Capella, thinking was still so much alive that these Platonic bodies were seen when people thought. Today we have to name them. The inner light of thinking gradually dimmed down to such an extent that—in the 5th, 6th, 7th, 8th centuries after Christ—names and words replaced what was formerly the glimmering fluorescence of our thought life.

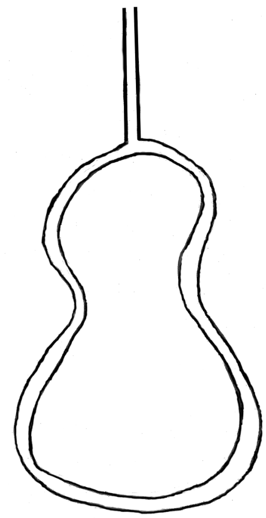


You can hardly imagine how it was to think—round about the second, third, fourth century before Christ. The light which was shining around and within our heads was so alive, so wonderful, that within this light the ideas of the world incarnated. But this light had to dim, and it did so. It was buried. It was buried from the 4th century onwards. Today as it were—understand what I mean—we have become a kind of pianola: a self-playing piano. Only gradually does renewal come about. With the death of the light of thought, the world opened itself to our sense-impressions; and we looked out and started to behold the kingdoms of nature. Minerals, plants, animals, man—in ever-increasing number—until the number of observations and phenomena is so overwhelmingly great that today no one is able to survey even a ten-thousandth of what is known. This is what I meant. Geometry, Arithmetic and so on have taken on a garment.

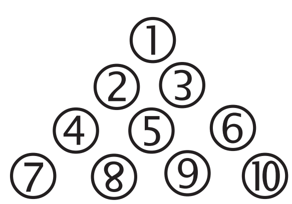
What was for Boethius still Geometry is for us *Mineralogy*. We behold the crystals; we see the stones. And if we see it rightly, properly, we measure the geometry of the light within space—in pyrites, in fluorites, in salt, in any other kind of crystal. In the crystal formation, in the crystalline formation, even in the rock formation. Then we have to introduce higher mathematics—calculus and so on; this is Geometry.

Arithmetic has taken on a most beautiful garment, and this is *Botany*. How the plants grow, open their leaves, open their flowers, form their fruits. You remember I told you in our last Conference that Geometry is space, Arithmetic is time. The plant grows in time through its various organs from the seed to fruit—and thereby explains, reveals, its arithmetical existence. Whether it is the order of the roses wherein the number 5 is revealed, or the order of the lilies where the number 6 is revealed, or others where 4 or 2 is revealed—it is nothing else but arithmetic. We know that the nodes in the stem follow exactly the logarithm; they can also follow an arithmetical or a geometrical law. It is arithmetic—it is number—which has color, which reveals itself: There is no other possibility in form.

Music is in the *Animal Kingdom*. Do you know that every animal body is a musical instrument—which is either played from outside, with the invertebrate animals, or from within, with the vertebrates? If you look from above at the form of one of the higher mammals like the horse or the donkey or the cow, you will see it is nothing but a stringed instrument.



You must only have a certain understanding for what music is. All the muscles, all the nerves, everything is arranged accordingly. We are not going to speak about this now—perhaps in a year or two we could take up this subject in detail. Number in botany reveals itself in the flowers, in the leaves, in the stem—by the power of light. Number in animals reveals itself by the power of sound, and in the highest animals it even begins to sound from within (the moo of a cow or the strange whinnying of a horse has something of the quality of a number). Astronomy has turned into Anthropology. The study of man from embryology up to anthropology, from psychology right into physiology is the Astronomy of today. So the study of the Quadrivium today would be the study of the four kingdoms of nature. This is what we have first to consider. After this, we should take the Trivium—the study of the Word: But how to look at this would be another question.



1 2 3 4 5 6 7

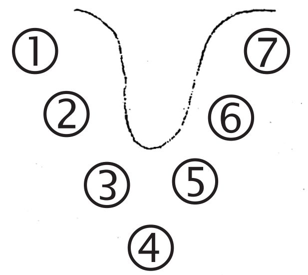
8 9 10 11

I would again like to point out that there are two forms of number: The quality of number and the number of number. If I have 1 and 2 and 3 and 4 and 5 and 6 and 7 and 8 and 9 and 10 (there is nothing behind such a form—I only try to picture it), each number has its own place, but I do justice to it only if I make each number an independent being. Quite different is it if I write down another row of numbers, in which it does not matter how they are written.

The one sequence of numbers we cannot count—it would be utterly wrong to attempt it. In the other group we can not only count but can even add 2 and 3 together to make 5: But this has nothing to do with threehood or fivehood or sevenhood or fourhood. This brings us near to what the quality of numbers might mean for us. I would like to read a few indications of Rudolf Steiner about the numbers 7 and 12. In the eighth lecture of the cycle "Der Orient im Lichte des Okzidents—Die Kinder des Luzifer und Die Brüder Christi" ("The East in the Light of the West—the Children of Lucifer and the Brethren of Christ," München, August 30, 1909), he says:

> *Nun, alles was zeitlich angeordnet ist, das ist nach dem Mass und nach der Natur der Siebenzahl angeordnet. Wir beherrschen den Weltenwerdegang, wenn wir die Zahl Sieben zugrunde legen, indem wir sagen: Saturn, Sonne, Mond, Erde—Mars, Merkur—Jupiter, Venus und Vulkan—indem wir wiederum zum Beispiel hier sieben Zeiträume haben. In allem, was der Zeit unterliegt, finden wir uns zurecht, wenn wir die Zahl Sieben anwenden. Daher werden wir überall da, wo wir in die Zeit hineingefuhrt werden, zur Siebenzahl geführt werden. Alle die Kollegien und Logen, welche aus dem Raume hinaus in das Zeitliche hineinführen, wenn sie zum Ueberweltlichen fuhren, unterliegen eben der Siebenzahl. Der Siebenzahl unterliegen die heiligen Rishis, der Siebenzahl unterliegen die anderen heiligen Lehrer der Völker bis herein zu den sieben Weisen Griechenlands. Die Grundzahl des Raumes ist die Zwölf und indem die Zeit herausfliesst in den Raum, wird sie zur Offenbarung durch zwölf.*
>
> Everything which is ordered according to Time is—after the measure in nature—ordered according to the sevenhood (or the number seven). We, as it were, rule the becoming of the world, if we put into the background of all existence the number seven. We, for instance, say: Saturn, Sun, Moon, Earth, Jupiter, Venus, Vulcan—these are the seven great globes wherein the earth develops. Everything which underlies Time, we have to order according to the Seven. Therefore wherever we have something to do with Time, we will need the Seven. All the colleges and lodges which lead out of Space into Time are ordered according to the seven. The number Seven lies behind the Seven Rishis; the number Seven lies behind all the great teachers of peoples, up to the Seven Wise Men of Greece.

A first inkling of the number Seven is given. Steiner always drew the time of evolution—with the seven epochs—in this form, which in itself helps our thoughts and ideas to turn to what we can call Sevenhood.



Then Steiner goes on to say that the fundamental number of space is 12:

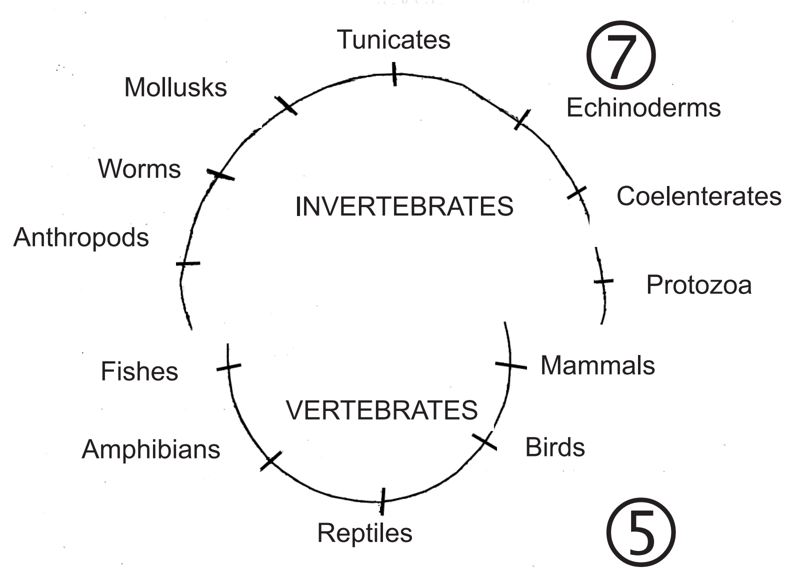
> *Daher herrscht die Zwölf da, wo die Zeit ausfliesst in den Raum. Zwölf Stämme haben wir in Israel, zwölf Apostel in dem Augenblick, wo der Christus, der sich vorher in der Zeit geoffenbart hatte, herausfliesst in dem Raum. Was in der Zeit ist, ist hintereinander. Was daher von dem Raume in die Zeit hinführt zu den Göttern des luziferischen Reiches, führt in die Siebenzahl hinein. Wollen wir etwas seiner Wesenheit nach charakterisieren in diesem Reiche, so finden wir diese Wesenheit, wenn wir das zu Erforschende auf seine Vaterschaft zurückführen.*
>
> If Time flows out into Space, it becomes the revelation of the 12. The Seven is the flow of time. But if this flow of time (in the rhythm of Seven) enters space, it cannot remain seven: it reveals itself in Twelve. The Israelites had twelve tribes. When Christ appeared on earth, there appeared around Him the Twelve Apostles. All this means that the Twelve reveals itself in the Logos—when the flow of Time enters the Space which is Light.

If we begin to feel our way into such a statement, we will understand that the Seven is always present, the *Gegenwart*. The Seven is always the Becoming. Saturn, Sun and Moon belong to the past—but also to the present. The whole world of Saturn, Sun and Moon that we carry in us, as far as this is time, reveals the Sevenhood, which means the present. We reveal it, the world reveals it, everything. If we let this present flow into space—look at the revelation of what has happened—it reveals itself in the Twelve. This is the Past—"*Es ist gewesen*"; this is Twelve—it IS. But the 'in the becoming'—the happening—this is Seven. The 12 is the number which brings to a conclusion what started with the One—not with the '1' but with the 1 [circled 1]. The 1 went into time. Time took up the 1, made it into 2, 3, 4, and so on; and when it was concluded—when it had entered space and become—then it was 12. So the end of the 1 is the 12: therefore the 12—even for ordinary arithmetical conundra—has such glorious possibilities of being divisible by 2, by 3, by 4, by 6. You can already feel how Grammatica and so on has intimately to do with all this. To understand it we must, as it were, divide, disintegrate, specialize: but behind stand Word and Number.

Now listen: Seven is Time, Twelve is Space. When Time flows into Space, it reveals itself from the Seven into the Twelve (we will soon have an example of this). But the flow itself—the process whereby Time enters Space to reveal itself in it—is the Five. Not because seven and five is twelve; this has only in the last resort to do with it. But the flowing from time into space is one of the revelations of the quality of the Five.

7 5 12

The greatest revelation of the 1 and the 12 (you understand now what I mean) is, of course, the zodiac: and the zodiac emanates formative powers out of time into space. We owe it entirely to Rudolf Steiner (in no form of schools of occultism do we find anything of it) that he revealed for the first time the various forces flowing out of the 12-hood into space, which are revealed by the 12 orders of the animal kingdom. If I would draw it properly, I would have to show the seven above and the five below, together making the twelve: and they are all one—because it is the animal kingdom. Nothing else. Here are the seven formative forces which reveal themselves from the oneness of the twelve: the protozoa, coelenterates, echinoderms, tunicates, mollusks, worms and arthropods. And here the fishes, amphibiae, reptiles, birds and mammals.



I will for the moment only characterize the seven invertebrate forms—to start you wondering which number is related to each of the seven classes.

The *Protozoa* are unicellular nothingness—hardly animals, or even plants: but they have certain qualities of life—they split into two, procreate, they breathe and have a certain amount of metabolism, they take in substances and shape substances. The cell, during the last fifteen years of scientific investigation, has become the most complicated organism imaginable. A single cell consists of varied and numerous organs, yet in itself it is a mass of dependent-independent nothingness. One can likewise imagine 30,000 or 60,000 watchers at a football match as unicellular nothingness; one can rightly describe it as a mass—unproductive yet reproducing.

The *Coelenterates* are still not quite animals, at least in certain parts of their existence: most of them are fixed to the bed of the ocean river (there are both fresh-water and salt-water polyps). They have not yet a distinct form, though they are usually round—with a few tentacles sprouting, shedding their ovums around them. Sometimes, out of one of these, an entirely different form grows and unfolds into a jellyfish—a living parachute, not in the air but in the water, which hardly swims but hovers through the water. There are varied specific organs arranged around the rim of this parachute: the first nerve-fibers go along the rim, a mouth (which is at the same time the anus) swallows and pumps out fluid. But there is already free movement. And this polarity between the fixed polyp and the hovering, swimming, moving jellyfish is, so to speak, the archetypal form of what the coelenterate reveals.

The *Echinoderms*—singularly—have five as the geometrical form of their existence: whether they are in fact round or a five-pointed star or whatever else between these two forms. They now start a kind of breathing, not in air but in water; yet they are lazy in their movements. A hollow space is at the same time mouth and anus, but the fundamentals of an excretory system (which later becomes kidney) is already present. The Echinoderms, together with mollusks, are the oldest creatures. In the becoming of our earth they reach right back into the palaeozoicum; they were the first ones to become hard substance; usually lime is their substance.

The *Tunicates*, in their central position, have very few orders, families and species, but they are the first ones to develop a kind of spine, a so-called corda. They are not vertebrates, but some modern scientists consider that the tunicates present the archetypal forms out of which all vertebrates unfold.

The *Mollusks* comprise the mussels, the snails, the octopus—usually a hard form covering a very soft body (think of an oyster, tightly held together), and in the snails this hard form develops into the most intricate spirals. The number of different mussels is almost incredible: they have tens of thousands of forms, from hundred-coil mussels to flat mussels. And their varied colors and formations are also repeated in the forms of mammals.

The *Worms*, of all kinds and descriptions, have hundreds of thousands of species. In the worms the main principle is repetition in the building of the body. One segment follows another, from head to tail, and sometimes you can hardly distinguish which end is which. If you cut the earthworm into four or five parts, each part will develop a head in front and a tail behind—because the difference is by no means great. Somewhat as a plant grows from leaf to leaf, so a worm repeats itself.

In the *Arthropods* we at last find a distinction between a front (the head), a back (the body) and a central part (a chest which carries the limbs). Only in the arthropods, after such an enormous way from one to seven, does a first form of threefoldness appear.

H.v.W: How can we imagine the way from 7 to 12?

Dr. König: Over the threshold of the 5 the 7 enters the 12; 7—via 5—to 12. As soon as the 7-hood, which is becoming, enters the 12-hood, it is already done.

J.S.: Could you give an example?

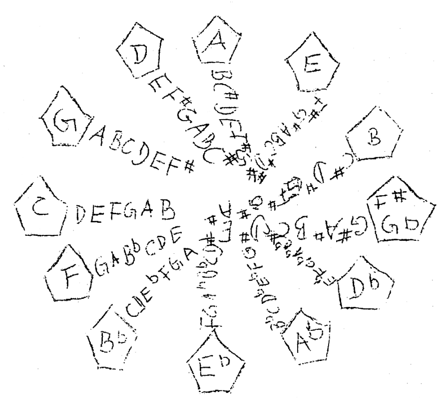
Dr. König: You can't. You must think it.

M.M.: Can one imagine the 5 as a factor to come to the 12-foldness?

Dr. König: Rather call it a 5-hood, threshold towards the 12. Go from the 7 to the 12 through the dodecahedron.

C.A.L.: In music you can find a concrete example. The Five stands as bridge between the Seven and the Twelve: this can be observed in music especially. The pentatonic system forms a bridge between the strongly anchored diatonic (seven-tone) scales and the dodecaphonic music-making that deliberately shows no center of relation.

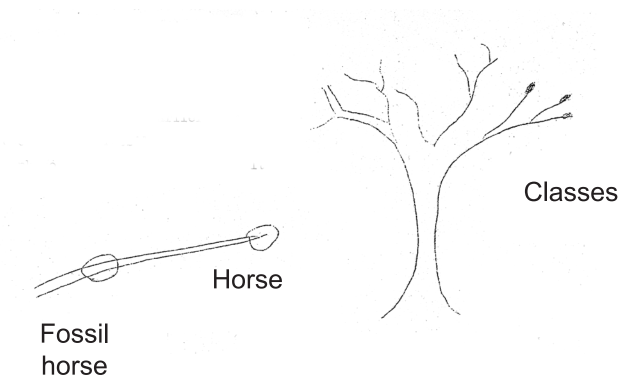
Another example is the fact that each of the twelve tones of our present system (as represented by the circle of fifths) can be seen to have five tonal possibilities. F sharp can be experienced as G flat, A treble-flat, E double-sharp and as a tone of its own standing. In a similar way can G become G sharp, G double-sharp (sounds like A), G flat or G double-flat (sounds like F). E. Bindel put it that each tone is a tone constellation and bears five aspects within its own house. Thus we can see that diatonic scales (seven notes) spring from each of the twelve tones like out of a pentagon-house. Here the Five again becomes the threshold between the Twelve and the Seven.



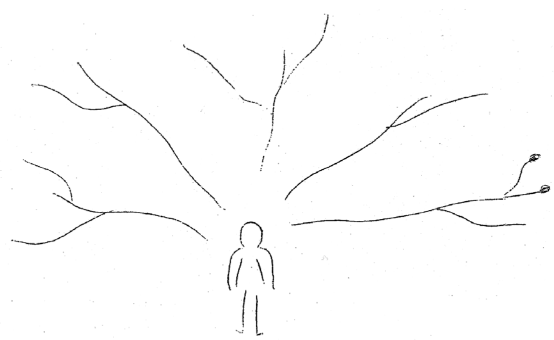
Dr. König: The seven ways of the planets, each one with its own 5-fold expression within the 12-hood.

G.C.: Can one discuss this in terms of evolution?

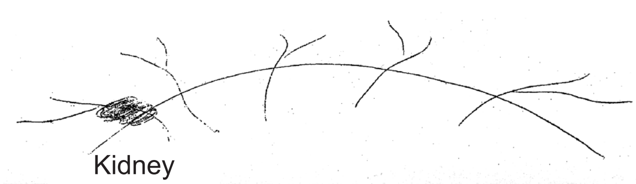
Dr. König: The difficulty of zoology and anthropology today is that science finds itself in the following quandary. Usually one speaks of a tree. This branches out, then the twigs do so. This gives an evolutionary image of the tree. Try to imagine zoology with all the many species from the times gone by right until our time. There are seven invertebrate classes, and each one has its own origin. Certain animals are known in their earlier stages of evolution only as fossils. Where is their link to the trunk of the tree? It is truly missing.



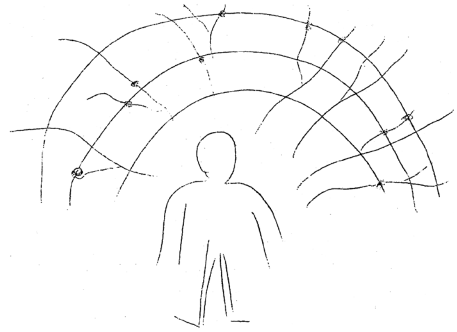
The origin of the species is the archetypal image of Man; the tree should therefore look like this:



In arthropods the kidneys are very simple in form and function. The organs show definite evolutionary steps on their way through the many animal species. We find the seven organs moving through the twelve classes of the animals



The seven organs (heart, lungs, liver, kidneys, stomach, spleen, gall) have developed in such a way. The brain is one; it was—it is twelve. Revolutionists speak of the 7 organs; evolutionists are rightly concerned with the twelve forms, groups of animals.



H.C.V.: Is it so that space is created in the growing process?

Dr. König: Such concepts will be rightly approached only when we begin to learn that space and time are not units, but that there are many different spaces and times. The time for sleeping and waking is an entirely different one from the time of breathing. We breathe 25,920 times per day. In a lifetime of 72 years we sleep and wake 25,920 times. There are entirely different times, and become one only because they meet. At the intersection of different times arises consciousness. The crossing-places are consciousness-forming. These are three spaces, the one of light, the one of sound, and the one of touch. What for the plant is a year for us is a day in its growing and withering away. For animals it might be a week.

H.C.V.: Is it right to think that our understanding of the inorganic world is illumined by the 12 and the organic world under the 7-hood?

Dr. König: Under the 12-hood you will understand the form of musical instruments. The 12 is the one. It needs a 13 and that is the one.

Yesterday we embarked on this almost world-wide question of the quality of numbers.

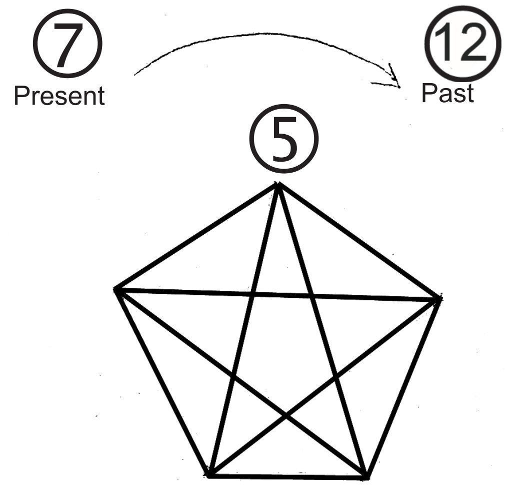
We started to concern ourselves with the essentials of numbers, and we began to prod our way towards a first understanding of what all this means. I think I made it clear that if we speak of a 2, a 3, a 9, we by no means mean 3, 4, 5, 6, 8, 9: what we are referring to—and this would have to become more and more intimate knowledge—is a kind of beinghood—a five, a nine, a three. We would have to imagine that here we begin to meet the helpers of the Architect of creation; and that helpers—the two, the eight, the four—try to form the design in such a way that creation is filled. In just such words we can trace the earliest essential in the search of Masonry—which true Freemasonry still strives for: to learn to understand the secret and the mysterious working of numbers. Today cheap occultism plays with numbers. One can't be too careful to avoid this: the search for the so-called mystery of numbers is beset with traps. Thousands of books are written about it, and among ten thousand pages you may find one sentence worth considering. Among ourselves, too, I remember friends who were counting days backward or forward, or looking at the horoscope and finding the angle of this or that planet 21 years ago, 18 years ago, 9 months ago. And of course we can find concordance in everything if we have all the numbers 1 to 12 at our disposal. Danger inheres in this playing with the mystery of numbers, this trying to find relationships in time and space, especially regarding one's own destiny. Everywhere the tempters (tiny little devils) look through and grin, when the threads of untidy thinking make one appear as the center of the cosmos.

Music contains a tremendous amount of this play—and it has no meaning. As long as you leave it as a piece of art, it is beauty; as soon as you try to pin it down and explain why this or that motif or melody of harmony or rhythm was used in this or that place, you do something which you will understand shortly when we speak about the number 5. This kind of relation—beautiful as it is, intricate as it is—does not concern us at the moment; we want, as it were, to go behind it. This is the beginning.

Yesterday we took up, at the hand of one of Rudolf Steiner's indications, the number 7 and the number 12. We tried to relate ourselves to the fact that, wherever seven appears, there we meet time. We meet development. We meet something which we called the ever-being-present, which is becoming. Whereas when we meet the twelve, we are in space: and, in the special sense, when we speak of space we mean all that is past. You see how we have to learn to understand each other; to speak geometrically about three-dimensioned, two-dimensioned, four-dimensioned space is subtly different from speaking of ‘space’ in connection with the number twelve. When we say twelve we know it is finished. Past. Created. In this way words gradually assume again more than replacement for what is meant. They assume symbol and meaning. Symbol and meaning which can be grasped not only by human thinking, but by the whole of man—as a thinking, feeling and willing being in unity. We connect with seven and twelve a number of experiences and phenomena. But if we would just express the number seven by a heptagon or the number twelve by a dodecagon, we would by no means come near to their beinghood. Though no doubt we would somewhere meet the shadow of sevenhood, the shadow of twelvehood; we would not meet the beinghood in the shadow, but beyond. If we stand with wonder and awe in front of the twelvehood, we can learn to see the whole of the zodiac. It is difficult for human thinking to behold a twelvehood—it needs years and years of daily exercises to be able to grasp in one image all twelve signs of the zodiac—because we are not so far in our thinking as to embrace the twelvehood. It is easier—because here our feelings and sentiments and rhythms are much more engaged—to live sevenhood: to live in the becoming of Saturn and Sun and Moon and Earth and Jupiter and Venus and Vulcan. How different the twelve senses, the twelve classes of animals, the twelve consonants!

Do you see what it means to take a first glance into the twelvehood? This means to look from the sense of touch over all the other eleven senses, right into the sense of ego, and have them all present. Beings are categories of thinking which we have to achieve more and more: without this a real understanding of Man and the World is not possible. The seven planets are a category of sevenhood: not one or the other (this would be an artificial experiment if we see the sevenhood as a whole), but all the seven moving around. And if we every day try to imagine the 12-hood of the zodiac and the ever-moving 7-hood of the planets, we will gradually learn that they begin to sing. And that in this singing and harmonizing, motifs sound—which the 7-hood plays on the 12-hood as its instrument. The instrument is made, is finished, is space; whereas the 7-hood is the ever-moving, becoming present. Whether it sounds in tones or days or colors or tastes or vowels is secondary. In this way, and I think only this way, gates begin to open to what we might call the Masonic understanding of the universe, the Masonic knowledge—if you take knowledge for the whole of man, and not only for his head—of quality of numbers.

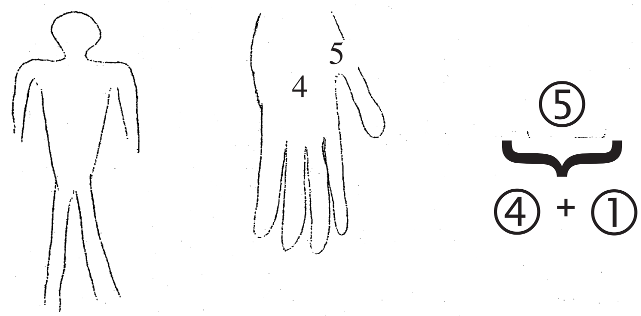
As we have said, in order that the 7 can reach the 12, in order that the present can grow into the past, we need the 5. If we would now imagine that this 5 is the pentagon, we would again just see the shadow of what is meant.



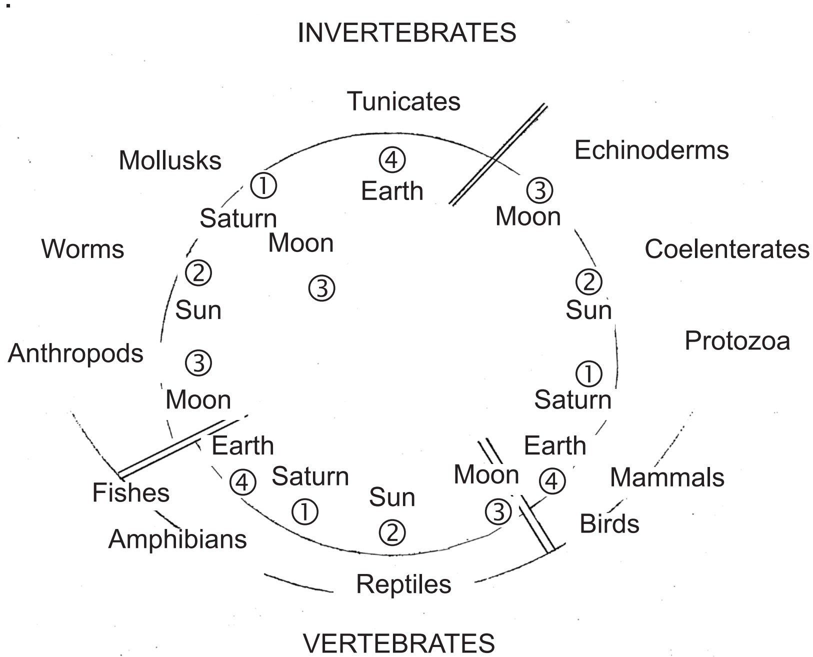
But what do we meet if we look instead at the human being with the 5 fingers—by no means a pentagon, yet somewhere it is? How can we understand it? Rudolf Steiner once gave a lecture on numbers (on the 14th September, 1907, in Stuttgart) and said:

> *Fünf is die Zahl des Bösen. Beobachten Sie einen Menschen. Er hat sich entwickelt, ist eine Vierheit geworden, Aber auf der Erde selbst tritt zu ihm das fünfte Glied, das Geistselbst. Wäre er nur eine Vierheit geblieben, so würde er von oben, von Gott dirigiert worden sein immer zum Guten. Er hat auf der Erde die Keimanlage erhalten zum Mars, damit hat er die Möglichkeit erhalten das Böse zu tun, dadurch ist er aber auch selbstständig geworden.Ueberall, wo ein Böses auftreten kann, das auch sich selbst verderblich wird, da ist eine Fünfheit im Spiel. An einem Solchen Beispiel können wir sehen, wie überall da, wo die fünf uns entgegentritt, die Berechtigung sich ergibt, von einem Bösen in irgend einem Sinn zu reden.*
>
> Five is the number of evil. Behold man: he has developed into fourfoldness. But here on earth the 5th member comes to him, which is Manas,[7] the Spirit-Self. Had he remained a fourfold-hood, he would have been directed from above and God would have made him to remain within the sphere of the Good. But here on earth he has received the seed of Mars, and with this the possibility to do evil. But at the same time he has become a self. Everywhere where evil appears, it is a five-hood which is present. From such an example we can see that wherever the 5 appears, there also appears the necessity to speak of evil.

This is a most important indication—but how can one understand it? Man is made, as it were, to be four—physical, etheric, astral body and ego—but here on earth a fifth is given to him. Let us imagine a man very simply: can you understand that it is this form? And now we see beside it our hand—with the little finger, the second, the third, the fourth. Which is the difference between these two?



This is 4 and this is 4: but now comes the fifth. After the four fingers comes the opening up towards the thumb. The one is added to the four. And here—with the 5th—evil sets in. But one could also say that the moment the 5th appears, day-consciousness arises: and with day-consciousness, space comes into play. So we would have to say: 'To lead the seven into the twelve, we need the five: but the five now appears as the 4 + 1 in the creation.' And now imagine, always and continually, one breath and four harmonies, one breath and four heart-beats: in rhythm there is exactly the same as in the four. And it is the rhythm in us which continually (think of the "Foundation Stone Meditation") leads time into space. Do you begin to understand? You can say, 'This is 5,' and you will be able to deduce many interesting things with triangles and so on. But how much more it means when we discover that four heartbeats and one in- and ex-haling is 5 [circled]. This is the gate into birth and the gate into death. It is the fate from time into space; from 7 [circled] into 12 [circled]. It is the form which shows that we are here on earth. This is the first thing I wanted to give you, as an example of how I imagine we should set about trying to behold number.



And now let us return to yesterday's 12-hood of the twelve classes of the animals—the seven invertebrates and the five vertebrates. The protozoa, coelenterates, echinoderms, tunicates, mollusks, worms, arthropods; the fishes, amphibiae, reptiles, birds and mammals. I hope you haven't been thinking that we have the secret numbers like this in the 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12! We will have to go carefully in order not to fall into one of the traps which lie waiting for those who deal with numbers. Let us try to imagine the protozoa, coelenterates, echinoderms (5-pointed stars), tunicates (with the first spine-like chorda dorsalis), mollusks, worms, arthropods (butterflies, spiders, bees, beetles—this millionfold world of small images). And then let us make a huge jump to the vertebrate: the fishes, amphibiae, reptiles, birds and mammals. With this we begin to glimpse the many forms, orders, colors, behavior, drives, instincts—the life which once made the whole earth a teeming Paradise, until man had to interfere. If we really see this, we will not only find a tremendous jump between arthropods and fishes, we will find a similar jump between echinoderms and tunicates. What does this mean? There are many answers; I try to give one. Into these 12 classes we can, so to speak, inscribe one point of evolution. We can go from Old Saturn, over Sun, to Moon, to Earth. And again from Saturn, over Sun to Moon and Earth. And once again from Saturn, over Sun, to Moon and Earth. What does this mean?

Three times the makers of the animal world begin to shape, in order to create in the end the form which the whole of this 4th evolution—the Earth—is striving for. We start with the protozoa—round cells: and we remember how Rudolf Steiner describes in *Occult Science* the first stages of Saturn, a unit of round bodies (balls, vesicles) of warmth, nothing else. But when Saturn-hood is left and the spirits which create it withdraw, the balls of warmth disintegrate, and that is the kingdom of the unicellular beings who throng existence—bacteria, protozoa. Uncountable—though each one contains the universe. Yet they are no more alive and real than a photograph: they are nothing but the photographic pictures of an evolution since Saturn. The coelenterates ARE already alive, and light is filling them, though there is yet, no form. The manifold colors reveal that Sun and air have unfolded out of the kingdom of unified warmth. With the echinoderms (starfish) a hardening process sets in. What is alive and colorful and shining in the coelenterates is fixed into a horny substance, consisting of a kind of outer skeleton. It is the Moon process. Recent moon-surface photographs show the larva as burnt-out hardened substance, similar to what Rudolf Steiner has described. With the step from Moon to Earth—from 3 to the 4 [circled]—tunicates begin. Tiny—but looking for something which is to come: the spine.

With the mollusks we begin, as it were, a second Saturn—in a different sphere. All is soft; only a shell is built around. And there are again the same uncountable billions and billions which we found in the protozoa. The mollusks close themselves off. The protozoa sometimes grow a manifold beautifully-shapes skeleton around their cell, and the mussels and snails continue to do this. But at this new level in evolution, the roundness is enhanced by the spiral. Just this enhancement of the round to the spiral is typical of the world of the mollusks. The worms, next, bring us again to the polyps. But now, at this new level, they are formed out of the number of segments. With the arthropods a fixed form is enclosed in a shell, an armor—and for the first time something appears which we might call limbs. They are perhaps just another kind of sensory organs—eyes on antennae, ears on a leg: but are these not limbs? In their wonder and beauty the arthropods are as complete as the echinoderms; there is nothing more to be added. A new jump, and the spine is developed: spine-bearing fishes swim in the water—and the hardness has gone, metamorphosed into the fin. It is again the 4th step.

In another new beginning, land is discovered: the amphibiae start to conquer the hardened ground, and begin to breathe. Now—in a peculiar way—the 5th really starts. If we draw four limbs in a pentagon, it is nothing but a frog; though it has already a spine, it is again as soft as the inside of a mussel or a snail. With the reptiles and snakes we meet—on the new level—worms and coelenterates: though now of course filled with earth, with spine. Then the birds—hardened, dried out. Nothing but spine-bearing insects or flying echinoderms. The higher insects also fly, and the lowest birds walk over the ground. Yet, like other 3rds, this too is incomplete—and we wait for the next jump, to the mammals. In what we meet here we can read the 3 x 4-fold attempt to bring into being what this earth is made for. They have, for the first time, four legs: and what do these legs do? They keep an equilibrium between the power of gravity and the light of levity. In the first attempt, with the tunicates, the spine is nothing but a way of light formed in the darkness. In the second attempt—the fishes—the spine is a ray of light. And now (the third time) legs make their appearance—in carnivora or beasts of prey, in ruminants or rodents: four legs on which to stand, to run, to hop, to jump—to negotiates the powers of gravity in the joy of overcoming it. Yet only man takes into the spine what tunicates and fishes have attempted. He—and with him all the anthropoids and other apes—puts forth his head as the 5th arm, and with it also develops the thumb. This he tries to make equal to the four, giving himself up to the powers of gravity in his feet. I don't say more. I need not this time go beyond the 4th to the 5th. Except to ask: Where does this come from? Where does it originate?

In answer I point to the protozoa, to the mollusks, to the amphibiae. They are the three forerunners—1, 1, 1 [all circled individually]—of what finally becomes the human head. A mussel (the oyster, for instance) is nothing but a primitive image of our head organization. If you feel yourself into a mussel or try to experiment with it, you will know as a whole they react to light: they are an eye. They react to sound: they are at the same time an ear. What in our head is specialized, in a mussel is still one. The protozoa are desolate, and mollusks are isolated; and even the amphibiae scarcely reach the earth: yet these are the three steps which have made our head. Dear friends, this is the 1 [circled] which is added to the 4 [circled] in order to make the 5 [circled].

Many more things should be said, but I have the impression we should close with this. In the afternoon we shall make a further attempt to bring these things for the first time into a certain reality. I hope I have achieved it.

A.L.: I want to refer to yesterday's lecture. In turning to the Animals we have approached the mystery of numbers through the garment of Music. Could we likewise seek for the sphere of botany through the garment of Arithmetic, the sphere of minerals through the garment of Geometry, and the sphere of anthropology through the garment of Astronomy?

Dr. König: It is possible, but it is a dangerous task. Grohmann, in his book on the plant, has tried to divide the plant kingdom according to the twelve signs of the zodiac. But it is questionable whether this was justified, and he himself would have liked to undo his work again. The plants are not as finished as the animals, and therefore to place their presence of seven in the past of twelve is difficult. All the more as Rudolf Steiner once clearly indicated that the whole plant kingdom consists of seven group souls, centered in the center of the earth. I would say that only the fruits can be ordered according to the 12. But there is in the plants something similar to the vertebrate and invertebrate animals—in the lilies and the roses. Which is something close to it. And again you find in the plants the unicellular non-entities—but also moss and the like—which you can correlate to the first creation, the first form of the invertebrate animals. But you would then have to put the roses with the (5) of the amphibiae and this is already artificial. But if you would start to speculate with the roses (this is not only the red roses and while and yellow roses, but a huge group of rossacae), you would place them here. And the rest is, in point of fact, the 7-foldness. With minerals I would not be able to say. Our friend Kolisko, who was a genius in such matters, once tried, but he has not achieved anything worthwhile. But with man it is simple. From head to feet (and also with the limbs)—from Ram to Fishes—the whole of the zodiac is man.

G.C.: Does the ordering of the animal classes conform with the normal ordering of zoology?

Dr. König: Yes, yes—more or less. Some scientists count the tunicates to the order of the vertebrates—taking all the vertebrates, including the tunicates, together and making one class into six sub-orders. Some zoologists think that worms belong to the arthropods, but more or less all the other classes are recognized in exactly the same way.

M.M.: You spoke this morning of the four legs of the mammals, and then you connected the 4 and the 1.

Dr. König: The 1 is the development of the head: and I went from the protozoas, of the mollusks, to the amphibiae—as the forebears of Saturn, Saturn, Saturn.

F.B.: Is this the way Rudolf Steiner indicated it for the teaching of the first study of man and animal?

Dr. König: Exactly, exactly. Most certainly. In the 1919 lectures (which we referred to during the Holy Nights in the lectures *Body, Soul, and Spirit in the Light of Reincarnation and Karma*—pp. 20, 30–31), Rudolf Steiner speaks about the head organization and the rest of the human body, clearly explaining that we can in fact only speak of evolution when referring to the head. The head is part of the animal kingdom—evolving out of it; the rest of the body does not, but is a new acquisition of the earth. And in this sense it was meant. I also hope, dear friends, that what I have said will not tempt you to think that first there were protozoas and then tunicates and then echinoderms. These are steps of unfolding, but by no means steps of evolution. As I explained yesterday, all the twelve orders or classes of the animal kingdom are formed out of 12 different directions and not one after another. We see them as one attempt here and another there, but they are parallel attempts.

I.R.: Otherwise they would not be 12.

Dr. König: That is the thing. If they were one after another, then it would be 7. So it is.

I.R.: You said the plant is not yet as finished as the animal.

Dr. König: No plants except trees have a hardness at the surface. With plants we cannot speak of being born and dying: a plant doesn't die—it cannot. A plant is continually developing—whether it is the 5th row of leaves or the 126th makes no difference: it is, and again it is. If a flower fades, a certain feeling arises in us, but this has nothing to do with dying.

I.R.: There is also a form element in the plant kingdom that results in seed, grain, etc.

Dr. König: There you are; you have already proved it! Wherever astrality meets, form becomes hard. Even in a bud. Only in the flower does a plant really reveal itself, whereas an animal reveals itself in its whole body.

T.J.W.: In a rather primitive and intellectual way, a non-anthroposophist might be tempted to see in the leaves of a plant the limbs. The petals of the flower usually conform to a certain number, and the rose conforms to its fivefoldness, even if it has many petals: but one does not find such a thing in the leaves, which really depends only on the time when one sees the plant. We couldn't imagine an animal having 3 legs when it is young, and 7 when it is older, and when it is still older 25, and yet we take that as normal for a plant. When I for the first time saw a one-year-old fir tree, I had no idea it was a fir tree. We usually recognize plants by their familiar forms—but the same plant may have different forms which we do not recognize. And that is not so for the animal.

F.B.: Can one in terms of evolution compare the seven orders of the plants with the seven organs in man?

Dr. König: I think one must; this will gradually become necessary. Don't ask me how, I don't know. From my acquaintance with the numbers, I have some ideas. But they are still only hazy. But you should try to imagine the seven orders of the plants with the seven days of the week. This would give you much more of the development of the plant, and from there you could go further.

S.B.: Could you say something about the Feeding of the Five Thousand?—with the five loaves and two fishes for five thousand people and the twelve baskets remaining?

Dr. König: One could definitely speak about this—also about the feeding of the four thousand and the parable of the Ten Virgins. Many such things. It is intimately connected, but it would go beyond our present realm.

I.R.: Alona mentioned the mineral kingdom. And you said it would not be possible to see it in this aspect. But do not the metals belong very much to the mineral kingdom?

Dr. König: The metals are seven; she was concerned with 12.

I.R.: Yes, but you spoke, in connection with the plant kingdom, of the 7 days; and it is obvious how the metals too are intimately connected with the 7 days. But it is strange to feel that what is most finished—the mineral kingdom—should reveal itself, with regard to number, as a process, as a presence.

Dr. König: The metals are not finished. The metals are still present. Therefore the one which is really alive is Mercury. And Rudolf Steiner once said it is quite wrong that not all the seven metals are still alive. They ought to be. And I do not think that it is justified to be so certain that the metals belong to the mineral kingdom. A copper sulphide or any kind of chemical compound, of course, is a mineral, but the metal itself is something else. You cannot call it (5) but you can call it (7).

C.A.L.: We tried yesterday evening to divide the concept from the symbol in the handling of numbers. For some of our children it is so difficult to establish that 4 is not 1, 2, 3, 4 but '4'—as a concept. Could you say something about the forming of the concept in us?

Dr. König: I would only say that I have not the impression we are born with the concept of numbers. We are definitely born with the concept for vowels and consonants: I don't think for numbers, we only gradually acquire it. In the 4th Post Atlantean time, through geometry, arithmetic, music, astronomy, man began to divine the concept of numbers. In our own epoch it begins to come to us out of the phenomena of man and nature. And I do not think that we should press our children to have the concept of number. As long as they can add and subtract and multiply and divide—and learn that two is bigger than one—it is enough. I have the impression here we go into the future—therefore it is so difficult to speak about. It is something which has to do with Manas and not with what has been. We begin, as it were, to draw on a store of our existence. We would also be wrong if we would think that the sense of life or the sense of movement gives us a concept of numbers. Not even the sense of equilibrium gives this. Though all counting, arithmetic, reckoning—a process—may come out of it. So I would see it.

H.C.V.: In this light I can understand a remark by Rudolf Steiner which always puzzled me: that with regard to our number concepts, nominalism is right.

Dr. König: You see! I didn't know this. That's the thing. We can't do differently. And what we do now is simply the attempt to start a new realism. And this is difficult, dear friends: every human being has a struggle to get over it.

I.R.: You said in "The Three Years" that the child, in developing the sense of thought, looks at the world through the windows of the words—and that we should accompany its experience with our feeling. Doesn't this now repeat itself in us in regard to a new sense?

Dr. König: Exactly. What we try to do is (in referring to yesterday) in the Quadrivium to achieve what we have got to achieve. We just try to open the first bit. One thing I would have to say. And this is I refer, of course, to thinking. In our willing—in our willing—the concept of numbers is present. Otherwise we could not make a single movement. And a real piece of art is, of course, nothing else but a picture of some of the number concept. But this is very difficult. And we have not got this concept unless we hammer through the phenomena or we just play about like the astrologists: Today is the 12th February. 12 and 2 (the second month) is 14—it is 2 x 7—and so on. And there meine Grossmutter war geboren.

I.R.: You said yesterday that the Middle Ages, in the Seven Liberal Arts, tried to repeat on the higher level the three steps of the first three years. Was that your own interpretation?

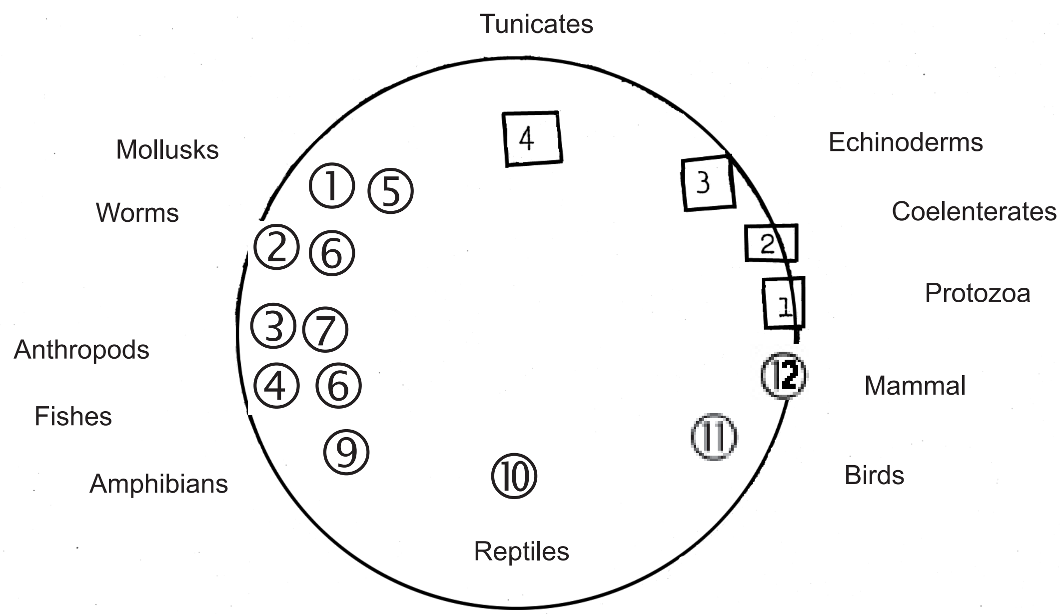
Dr. König: This was an interpretation of mine, entirely.

I.R.: Could you add anything to it? Is the Trivium so much connected with the language?

Dr. König: Yes, you see you have Dialectic, you have Thought, and you have Grammar. Now in Dialectic you move the limbs of your speech. In Rhetoric you move the speech of your speech—the melody, the harmony, the intonation. All that. Grammar is the logic of speech, of language.

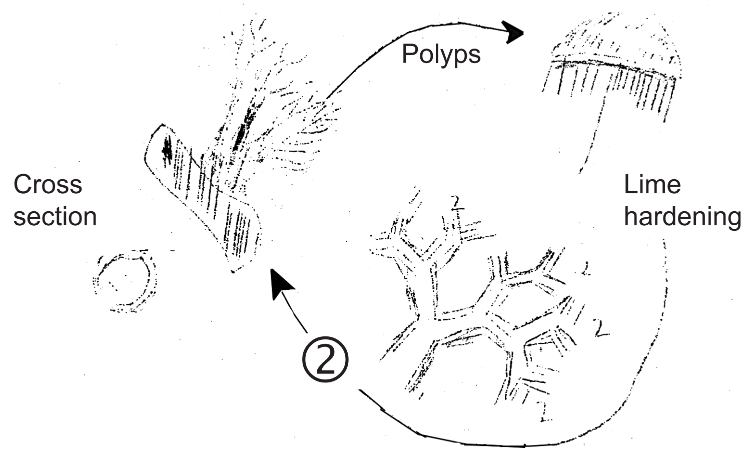
Conference 2 Seminar III

Once again I put down our central figure (otherwise we get lost)—and add to it the signs of the zodiac.

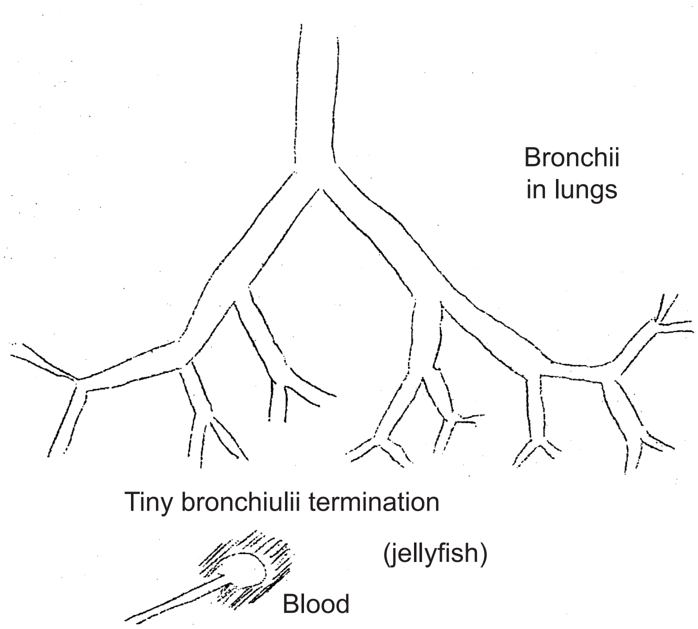


So we have the protozoas (in Crab), the coelenterates (Twins), echinoderms (Bull), tunicates (Ram). And then the mollusks (in Fishes), worms (Aquarius), arthropods (Capricorn), and fishes (Archer). Followed by amphibiae (Scorpio), reptiles (Scales), birds (Virgin), and mammals (Lion). Ample possibilities to speculate—and if you have to give lectures, you would make a tremendous impression on the unlearned audience. Don't do it: you would certainly go very wrong.

This morning we tried to understand that this is not only twelve, but 3 x 4. And that we find 3 times 4 not in evolution but in unfolding—and each time something higher is achieved. Now we have already spoken about the (1): the unicellulars are, as it were, left divided and becoming—an uncountable number of non-entities. The processes which we follow up in arithmetic (as I pointed out in our 2nd seminar) stem out of the wellspring of numbers: the protozoas—the unicellulars—progress in a continuous splitting up: 1, 2, 4, 8, 16, 32, 64, 128. And this splitting up leads—I only indicate—to a form unlike that of any other animal class or plant: it leads to separate single non-entities. But non-entities which are continually adding to themselves.



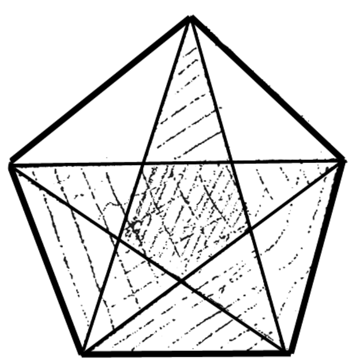
At stage (2) we can draw a polyp, something like this, sitting on a river- or sea-bed. Its unicellular body consists of two layers, an ectoderm (the 'skin') and an endoderm (the intestine); and on top are numerous tentacles of different lengths—beautifully shaped and colored, sometimes like flowers—which are in continuous movement, catching the prey and taking it in. But at times a bud occurs, which grows and creates a baby polyp—and this in turn creates another one—and so one. You may once have had the chance to see coral reefs—blue, red, rose, all colors—by the seashore. They were once coelenterates, which branched forth in their tens of thousands; and when this then hardened with lime and calcium and other substances, they became enormous rocks, reefs and ocean mountains. What in the unicellulars remains a single mass, here becomes a kind of growing animal/plant or plant/animal (in German they are called Pflanzentiere; in English sea-anemones—because something like a flower can be experienced), which sits on the ground and is only mobile within itself.



If a sea-anemone or a polyp is tired of bringing forth little ones, a bud falls down and makes a 'volte face,' developing into the 'parachute of the ocean' (as I described it yesterday)—the jellyfish. It's the same. This wasn't known 150 years ago. From this, that one can derive; from that, this one can derive. And this is a circle.

It is very difficult to say when this happens—but it happens at times. Some of us may have seen such jellyfishes hovering through the ocean—and perhaps experienced the effects of the poisonous stinging varieties. A jellyfish out of water, however carefully extracted, is just a (scarcely edible!) jelly—yet in water its colors are beyond all description, and one has the impression there is already some sentiment/feeling in them. Both these are (2): but there is a tension between the one which is still growing (not yet animal but longing for it) and the other one, which is already animal. These are divinations for us of what (2) can entail. Perhaps you would understand if I say: As the (1) is the womb of adding (always going in one direction), the (2) is the womb of subtracting (being taken off—the order of life alternately growing and being left behind).

In the human being is one organ which brings the two together. In the lung the bronchus and the small bronchii and the little bronchioli, and the tiniest bronchioli little-ized (getting smaller and smaller) develop the same way. But in the end, when the tiniest little bronchioli is reached, a vesicle terminates it—and these are the millions of jellyfishes which we carry in us. Nothing else! And these jellyfishes swim, or are surrounded and suspended, in the blood—as the jellyfishes are suspended in the ocean. The lung brings together what would otherwise remain apart—but it takes time. Not until the amphibiae are created is the (2) united.



③

(Hard core and shell)

(5 merging triangles)

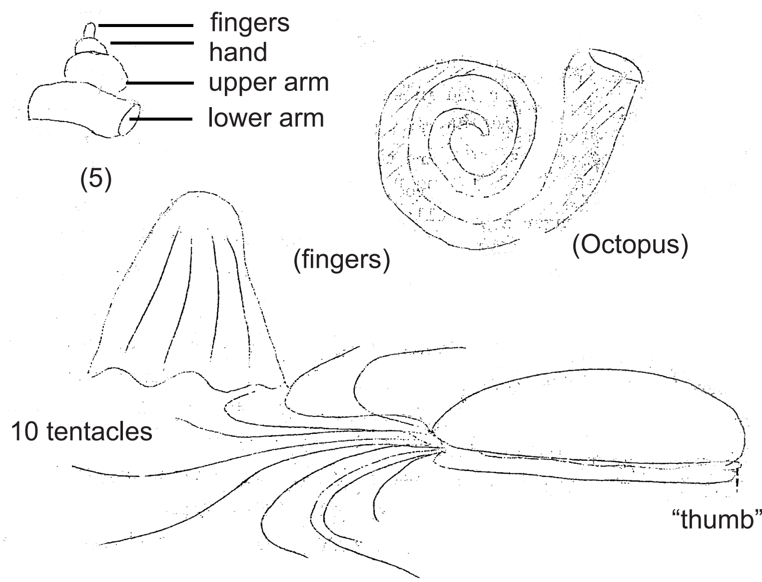
It would be easy to look at the echinoderms and say: 'the 5-pointed star—it's the (5).' But this would be quite unjust to the powers of nature. The echinoderms—starfishes—only pretend to represent the number of evil (which we learned about this morning). They are nothing else but a collection of triangles—as befits the number (3). Five triangles—five threes—merging together. Something entirely new meets us here, as a processuality (and we always have to point to the processuality). This is no longer the dichotomy—the growing of the lung, of the corals. Something now works together to bring about a kind of closure—perhaps even a uniting form: something which can appear as (5) or as the hexagon, or as the nonagon—as anything.

It is the (3) which makes it. (1) is the womb of adding; (2) the womb of subtracting; (3) is the womb of multiplying. Here 5 x 3 is multiplied, and creates something new. The whole kingdom of the echinoderms is built out of this power of 3. It has nothing to do with a triangle: it has to do with the triangles uniting, forming—and in the process developing a hard core and a shell.

With this we make a big jump to the (4). The tunicates for the first time reach out and create the corda dorsalis, the archetypal spine—which of course has intimately to do with the four. The powers which build the body are reduced; but in the end something appears which is almost nil, and yet is the center of everything around. To our (1) (2) (3) we can now add (4): divide. Dear friends, I cannot express it differently, but I would like you to try and follow what I consider as very first and exceedingly tentative indications. They are not more. In this small Conference we have taken up a new viewpoint: you can now begin to see how mathematics becomes zoology, and zoology is in fact mathematics.

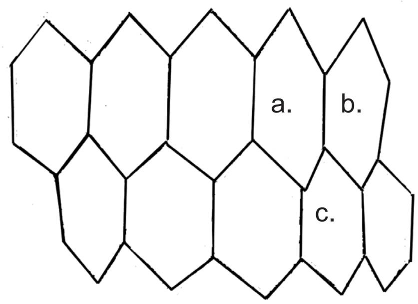
1 Adding
2 Subtracting
3 Multiplying
4 Dividing

Going one step further, we have again (1), (2), (3), and with the fishes (4). But this is not a simple repetition. We have already seen the mollusks as a second attempt of the protozoas, the worms, as a second attempt of the coelenterates, and so forth. This time we write here: (5), (6), (7) and (8). A pupil in an examination 200–300 years hence might have been asked: ‘How do you define the arthropods with regard to the quality of numbers?’ And he would have answered: ‘Sir, it is 3: the 3 and the 7.’ ‘Can you explain it?’ ‘Yes, sir.’ He would have known the meaning; we can have nothing but a few indications. … The mollusks develop three definite orders: the mussels, the snails and the whole group of decapods (Kopffüsser, octopus, squid, and so on).



Does this point to the (5) in the way we have imagined it? We have a great deal to learn if we are to come near to this. But the bone-work of the human arm (and it is similar with the limb of any mammal) can tell us something here. The humerus (the bone of the upper arm) is not straight. Neither are the two bones of the lower arm, or the 4 + 1 bones of the hand. Each one of these bones (and it is the same with the ribs) is modeled so that it builds a spiral. What above is in front, below is behind—forming a kind of a pillar which is in itself turned round. And even the finger is in the same way beautifully turned. Within each limb is hidden the spiral process. A shell is nothing but a rolled-in limb: fingers, lower arm, upper arm—this is all one. An octopus at rest lies with its infernal limbs rolled up in it: the same thing. It is simply your leg rolled round your head—nothing else (though this has, of course, nothing to do with the head). But the shells also present another form: our hands together, fingers pointing downwards. And where is the thumb? If we see it from the side, it is tightly closed; but a knob extends—the thumb which keeps the fingers together. It is always so. If we bang our cupped hands together and immediately clap hand to ear, it starts to sound. If I would use color, it would not only be beautiful, it would also help us to know where we are with all this.… And now, through the opening, appears a ‘head’—no real head, but a fan of tentacles: the cuttlefish. No longer in a shell: only a small plate within the body. But, around, the colors meet. (1) becomes (5). And in the same way (2) becomes (6), (3) becomes (7), (4) becomes (8). Another world opens up to us.

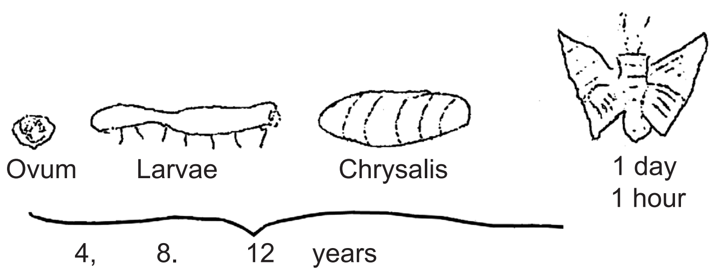
With the worms we reach the number (6), and I draw a hexagon. Like a honeycomb—but not only like a honeycomb; the human liver, dissected looks exactly the same—only a bit more complicated. It is not necessary to tell you about the circulation in the liver, and the production of gall, and so on. The (6) (the hexagon), is that form or principle of growth that is the enhancement of the (2). The (2) sprouts; the (6) fills the space in between. No other form fills space so tightly as the hexagon: nothing is left in between. It is 'ful-filled'—fully filled (*vollkommen durchdrungen*).



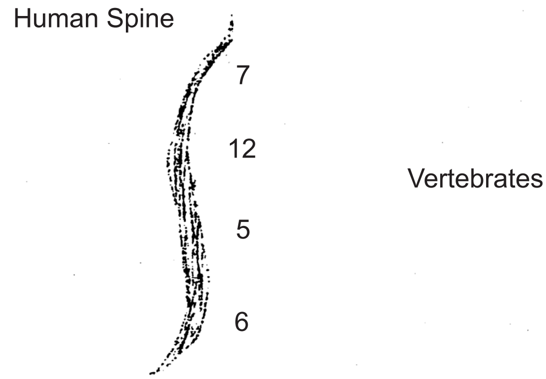
Liver

(beehive)

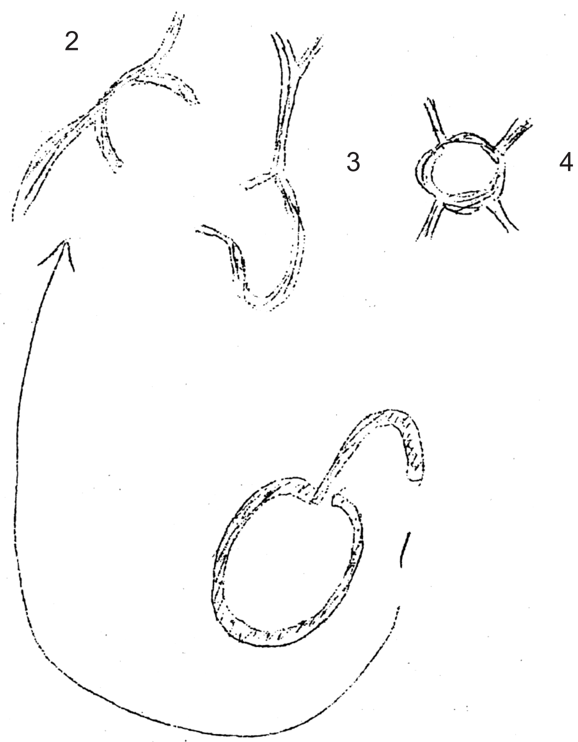
The (3) now becomes the (7)—and there weaves the arthropods. We have already seen that (3) is multiplying, is forming. But it does not form a sum: it forms a new unit. In the arthropods (as we said yesterday) head, chest and abdomen are for the first time formed out—though they are finished, hard, when they are formed. But in what way are they formed? Nowhere else in the whole animal kingdom is the wonder of metamorphosis so perfectly exemplified as in the insect and arthropods world. An ovum (egg) develops into a larva; the larva forms into a chrysalis; and out of the chrysalis arises the threefold image—which is, of course, a hand. From ovum to chrysalis may take four years, eight years, twelve years; the image may last one day, or even only one hour. This doesn't matter…. In the (7) we meet again the becoming. We meet this un-ending which ends in the (3) but is already finished.



Another big jump and we come to the fishes: we enhance the (4) and meet the (8). And now, with the vertebrates, we meet the wonder of number in an entirely new way. (1), (2), (3), (4) brought us through Saturn, Sun, Moon, Earth; and now—repeating this with (5), (6), (7), (8)—we again pass from Moon (arthropods) to Earth (fishes), from the third evolutionary stage to the fourth. The vertebrates create a new world: they are no longer instruments on which the world (in parts) is playing. The world mirrors itself in the eye of the insect; it is wrong to think that an insect look into the world. The insect is nothing but part of the world which meets itself in a 180-degree turn, from an outside skeleton into a spine inside. This is the way from the invertebrates to the vertebrates; and it is with the vertebrates that we can now start to count—because numbers, widely varying, now appear within. Some snakes or serpents have far more than a hundred vertebrae. Man has 7 in the neck, 12 in the chest, 5 in the lumbar region, 6 in the sacral region.



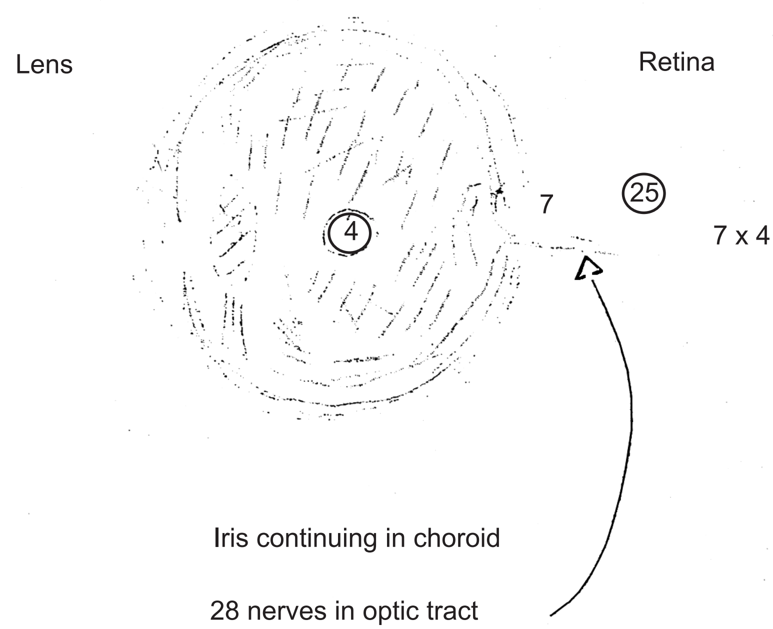
From (8) onwards—to (9), (10), (11), (12)—more or less the whole of the cosmos mirrors itself in one or another form within these realities. This is what we have to look for and to understand.



To close with I give you two examples and you will understand what I mean. Let us look into such an organ as the human heart (and I separate the four parts of the heart in order to make you see better). From above and below come two big veins: the superior and the inferior vena cava, which stream into the right auricle. From the right auricle they pass into the right ventricle, and thence into the ulterior pulmonary arteries. The blood-flow is rejuvenated in passing through the lung, and returns by way of the pulmonary veins into the left auricle.

From there the blood passes into the left ventricle, from where it goes out by way of the aorta. Now you see (2), (3), (4), (1)—and then through the aorta and the (2), (3), (4), (1) are repeated. And all this together in one organ. In a most intricate way a fourfold organ—two chambers and two auricles. Our (1) (2) (3) (4), (1) (2) (3) (4), (1) (2) (3) (4) goes right into the anatomy. This is not chance. The human existence is formed through by numbers.

After this, dear friends, I put to you a last image. (These are all images now, though very real ones.) I would like to show you the structure of the human eye. The human eye, you probably know, is a kind of ball—and whether we cut it through horizontally or vertically makes no difference because it is a globe. In front of this globe is the transparent cornea, and behind the cornea is the iris—in the center of which is the pupil, the round black spot through which we see into the world, through which we can see into the eye of somebody else. And the iris continues in a very interesting form, creating the second layer of the eye, the so-called choroids, which carries blood-vessels in order to supply the inside of the eye with nourishment. Behind the pupil and the iris we find the lens—a beautifully formed lentil, out of crystal-clear silica (but fluid silica): these are all crystals—cut through they would appear as hexagonal crystals, one next to the other, held by fibers of the choroids. Opposite the lens the optic nerve streams in and builds the layer of the retina. Inside is the substance of white jelly with star-like forms in it, filling out the whole remaining globe: the vitreous body. And now look! The cornea is (1), the iris (2), the lens (3), the vitreous (4), the retina (5), the choroids (6), and the sclerotic (7). And (1) is (7), and (2) is (6), and (3) is (5)—and in the center there remains the (4).



What I am telling you about during this Conference is already present in the human eye. In one of his wonderful remarks about the eye, Rudolf Steiner speaks of a remote little school of Rosicrucianism which prevailed right into the beginnings of the 19th century, where such a thing was still taught to the pupils. In fact many of the things I have tried to relate to you connect up with what was still known—in different words—to a very few in Europe and American 150–160 years ago. They also knew that if you follow up the optic nerve and how it enters the eye, you will find that it splits up into exactly 28 different nerve fibers—seven times four, which is a cycle of the moon. The sun goes through the 12-foldness, but if the light of the sun is reflected in each of the 12 signs, 2 ½ comes about in each—which makes 28 or 30, which is the number of these nerves and the number of our vertebrae. We could now begin to teach human anatomy: from the creation of the tunicates onwards we must start to think anew. But we must now close our session. Dear friends, these were nothing but indications. Nothing else. Nothing else.

Dr. König: I enjoy your questions and remarks, but I would be grateful if we could make this into a compact discussion.

I.R.: I couldn't follow with the numbers and the eye. Did you refer to the concept altogether? Or is it the single ones that have the character?

Dr. König: No—they are seven. I wanted to show you how the seven appears within the space.

I.R.: I thought you might be comparing it with the 3, where there is also the connection with the others.

Dr. König: Yes, you can do this, of course. But it isn't meant like this. It is the 7 within the space, as the closing image.

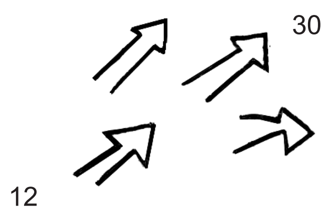
H.C.V.: When you say 2 ½: 28 or 30, how do you come to 2 ½?

Dr. König: If light shines, it can split. If the sunlight shines on the moon, it divides—and one ray, so to speak, turns into 2 ½ rays. Therefore 12 becomes 30 (which is 2 ½ times 12).

M.M.: I am not clear about the worms and the hexagon.

Dr. König: I supposed (though I said I did not) that you would know something about the life of the worms. The worms cling together as the hexagonal columns cling together. They like to live like this, to fill fully and fully fill the space. If you would cut this through, it would not be at all hexagonal. But this filling of space is an indication of the living tendency of the worms.

I.R.: Just because they are so space-filling, it is difficult to see the step in development from the worms to the insects.



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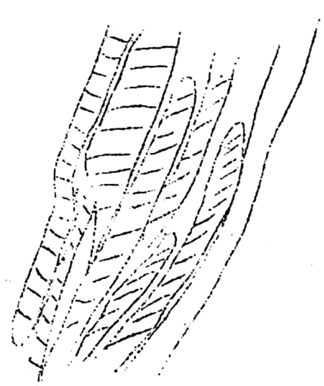

(Worms)

(space filling)

Dr. König: Follow up (I referred to it this morning) the 'embryology' (the early development) of the worms. They have exactly the same larvic forms as the arthropods, but then they turn into worms. There need not be any other connection; this is quite enough. Some zoologists even bring these two classes together on account of the similar larvic development—though otherwise it is an entirely different line of formation. I have the impression one should whenever possible substitute the word 'developing' for the word 'unfolding'—using 'developing' when evolution really takes place in the world of forms, and 'unfolding' when the forms are metamorphosed. (This would be a good exercise. I am not being narrow-minded, but it is good to be very conscious in our use of words.) And here in the one we have unfolding (arthropods), in the other developing (worms).

C.A.L.: Could you say anything about the number 10? I have the impression it is more than a special unit among the 12—because it is at the basis of the decimal system and creates a world on its own, with its own concepts.

Dr. König: We would have to take so much else into consideration that I have consciously avoided speaking about 9, 10 and 11. It would create still more complications in your minds than are there already. You are quite right that it is a special number—not only because it is 2 x 5, but also because it is the number in which the highest and the lowest, the most superficial and the deepest, are included. The 10 gives us the 10 Sephiroth and the 10 Categories—whereas with the serpent and the snakes we meet the 9. It contains as it were the whole universe, from about right down into Hell. If you would ask me the number of the Divina Commedia, I would say it is the number 10.

F.B.: Could you say anything about the connection in the Quadrivium between astronomy and anthropology?

Dr. König: Modern anthropology should be developed out of embryology. Rudolf Steiner has given us the task to do this: to relate embryology entirely to ethnology and to build up an astronomical method. When we are able to do this—we have a few ideas—we will have the metamorphosed astronomy we are striving for. To know Goethe's teaching that the skull is rounded like the heaven above us is something of the astronomy to come. Or to learn that the number of man's in- and exhalings per day is the same as the number of years the sun needs to go through the whole of the zodiac (25,920)—no animal has the same rhythm. Or to calculate the time of pregnancy as 10 lunar months. All this is astronomy. Man is, so to speak, built by the true astronomy.

M.S.: The twelvefoldness of the zodiac stands in space—but I had always believed that these animals existed after each other.

Dr. König: No, don't think that first were the protozoa and then the coelenterates, and so on. They came about out of different regions, not all at the same time, but by no means one after another.

M.S.: And even the higher animals were already there in their beginnings?

Dr. König: The fishes, for instance, were much earlier than the arthropods. Around the 5th and 6th epochs of Hyperboreas, when there was as yet nothing like earth or water, the fishes were already 'swimming' in the light. Don't relate it to the evolutionary tree; you would go entirely wrong.

J.S.: I believe you mean this to be taken as a whole. We should not select a particular number and remember what you have said about it, without relating it to everything you have said about all these numbers.

Dr. König: This would be good. But you should remember such things as '4 dividing,' and '3 multiplying.' With 3, remember the 5 points of the echinoderm. And remember the tension which creates the 2, and the complete division of the 1. Remember the 7 in time, the 12 as a whole again. Take these as very minor stepping-stones, and you will yourself begin to occupy your thoughts with the numbers.

J.S.: It is almost impossible, though, to take the figure of the pentagram and relate it to the 3 without convincing oneself of the whole entirety of what you have said. And the splitting of the coral into two, which is to obviously division or multiplication, one cannot relate to subtraction without seeing the whole.

Dr. König: Exactly: we have always to attempt this. And then we can, for instance, remember the 5 virgins and the other 5 virgins—and remind ourselves that the 5 is always something opposing, something to do with evil. Or think of the 5 loaves handed over, and the 2 fishes—which, going through man, created the new universe. Learn to live with such a thing. Learn to feel with it—but not feel in the ordinary sense of the word. The scientist of today looks at the form of the hand without relating the length of the four fingers to anything. And the ordinary man, who thinks in a scientific way, does likewise. And if I would turn my hand round and say: "So we stand in the three-dimensional space," both of them would say, "Rubbish!" But that is not rubbish—because some tentative observation comes in, and one begins to grasp an archetypal image. And there is nothing behind it: It is it. This is what matters, and this we should learn. And if we see how in the anthropoid ape the foot is formed like this – –, not like this /, we already see the opposing power going through. This is how I meant that we should approach the quality of number.

H.C.V.: I would like to express our gratitude for all you have given. I believe the greatest handicap in teaching arithmetic is to have only a number concept with which one cannot reach the souls of the children. You have opened up something that we can work with for years. You have led us to see the numbers again as lofty beings, and to start to equip ourselves in the background of our teaching with this new realism.

A.B.: It wasn't the lectures or seminars that we had here—it wasn't what you told us to take home, write out and remember. For me it was more like a process we went through. It wasn't that you lectured and applied it to a subject; you lectured and applied it to us. Those of us who are from the south had to approach this "initiation" under the threat of a railway strike—and to dare to come; and this wasn't without significance. When you arrived carrying the beautiful models of the Platonic solids, we might perhaps have divined why you brought them; you probably intended that we should now learn not only to apply our poor thinking, but to rise to Platonic vision and see ideas. So that we would not only talk about them, write them down and learn them by heart.

Dr. König: Thank you very much; you have clearly described it—from my point of view. Thank you, dear friends. It was very nice to be here.